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Early Time Ion Dynamics and Progress Towards Laser Cooling in an Ultracold Neutral Plasma

by

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Abstract

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The progress toward laser cooling an ultracold plasma is presented in this thesis. Ultracold neutral plasmas are created by photo-ionizing laser cooled Strontium atoms. Initially this system is very strongly coupled, however the ions rapidly heat up leaving the plasma on the borderline of the strongly coupled regime. To counteract the effects of this heating an attempt to laser cool the ions in the system is undertaken. However, from these experiments we discovered that velocity changing collisions occurring in the system prevented laser cooling. In order to determine the collisions rate, a collision model was developed to simulate the spectra of the plasma from optical pumping experiments. These experiments and simulations in fact demonstrate that collisions are occurring at a fast enough rate to prevent ion cooling on the time scale of our experiment.
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The completion of a journey is always bitter sweet; to make it to the end is always met with elation, however there is always a sense of loss at leaving behind what had become so familiar, no matter the trials that one experienced. The friends, the
memories, the laughs, the downs, the tedious jobs and the sense of accomplishment, all becomes wrapped in what was or used to be. But with every end comes a new beginning, a fresh start and new tasks to tackle so I look forward to what is to come knowing the things I have gained and learned here at Rice.

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## Contents

1 Introduction 1
   1.1 Creation of an Ultracold Neutral Strontium Plasma . . . . . . . . . . 4

2 Plasma Dynamics 7
   2.1 Disorder-Induced Heating . . . . . . . . . . . . . . . . . . . . . . . 7
   2.2 Electron Screening . . . . . . . . . . . . . . . . . . . . . . . . . . . 10
   2.3 Kinetic Energy Oscillations . . . . . . . . . . . . . . . . . . . . . . 11
   2.4 Plasma Expansion . . . . . . . . . . . . . . . . . . . . . . . . . . . . 13

3 Imaging and Spectroscopy of an Ultracold Neutral Plasma 16
   3.1 Other Plasma Diagnostic Methods . . . . . . . . . . . . . . . . . . . 16
   3.2 Absorption Imaging of an Ultracold Neutral Plasma . . . . . . . . . . 17
   3.3 Absorption Spectrum of an Ultracold Neutral Plasma . . . . . . . . . 20
   3.4 Fluorescence Imaging and Spectroscopy of an Ultracold Neutral Plasma 22

4 Second Harmonic Generation in an Optical Resonator 25
   4.1 Second Harmonic Generation . . . . . . . . . . . . . . . . . . . . . . 25
   4.2 Phase Matching . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 26
   4.3 Enhancement Cavity . . . . . . . . . . . . . . . . . . . . . . . . . . . 30
   4.4 Guassian Modes . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 32
   4.5 Modeling Guassian modes in an optical resonator . . . . . . . . . . . 33
   4.6 Longitudinal Modes . . . . . . . . . . . . . . . . . . . . . . . . . . . . 38
   4.7 Resonator Losses . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 39

5 Experimental Details 40
   5.1 Experimental Apparatus . . . . . . . . . . . . . . . . . . . . . . . . . 40
   5.2 Mode Matching . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 43
   5.3 Cavity Modes . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 44
   5.4 Phase Matching and Second Harmonic Power . . . . . . . . . . . . . 47
   5.5 Enhancement Cavity Feedback Electronics . . . . . . . . . . . . . . . 49
   5.6 Error-Signal . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 50
   5.7 Electronic Feedback Circuit . . . . . . . . . . . . . . . . . . . . . . . 52
   5.8 Procedure to lock the laser . . . . . . . . . . . . . . . . . . . . . . . 54
   5.9 Frequency Reference using Metrology . . . . . . . . . . . . . . . . . . 55

6 1092 nm Optical Repumping Laser 59
   6.1 Repumper Laser . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 59
   6.2 Optical Pumping Data . . . . . . . . . . . . . . . . . . . . . . . . . . 63
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 Laser Cooling Studies</td>
<td>65</td>
</tr>
<tr>
<td>7.1 Laser Cooling Ion Theory in an UNP</td>
<td>65</td>
</tr>
<tr>
<td>7.2 Scattering Force</td>
<td>65</td>
</tr>
<tr>
<td>7.3 Affects of Expansion and Recombination on Ion Cooling</td>
<td>67</td>
</tr>
<tr>
<td>7.4 One-Dimensional Laser Cooling of a Strontium Ion</td>
<td>71</td>
</tr>
<tr>
<td>7.5 Spectroscopy Studies of a Laser Cooled Plasma</td>
<td>73</td>
</tr>
<tr>
<td>8 Collision Rates</td>
<td>78</td>
</tr>
<tr>
<td>8.1 Collisions</td>
<td>82</td>
</tr>
<tr>
<td>8.2 Rate Equations</td>
<td>83</td>
</tr>
<tr>
<td>8.3 Three Level Strontium Ion Rate Equations</td>
<td>84</td>
</tr>
<tr>
<td>8.4 Determination of Thermalization Rate</td>
<td>86</td>
</tr>
<tr>
<td>9 Conclusion</td>
<td>90</td>
</tr>
<tr>
<td>A 422 nm Ring Cavity Autocad Drawings</td>
<td>91</td>
</tr>
<tr>
<td>B Code to Simulate Spectra</td>
<td>95</td>
</tr>
</tbody>
</table>
List of Figures

2.1 Disorder-Induced Heating ........................................ 8
2.2 Disorder-Induced Heating Density Studies ......................... 9
2.3 Electron Screening .............................................. 10
2.4 Kinetic Energy Oscillations ...................................... 11
2.5 Plasma Expansion ............................................... 12
3.1 Ion Energy levels ............................................... 17
3.2 Absorption Image ................................................ 18
3.3 Optical Depth .................................................. 19
3.4 Absorption Spectrum of ultracold neutral plasmas ............... 21
3.5 Schematic for Fluorescence imaging .............................. 22
4.1 Traditional Phase Matching ...................................... 27
4.2 Modern Phase Matching ........................................ 28
4.3 Second Harmonic Power in Crystals ................................ 29
4.4 Ring Optical Resonator .......................................... 30
4.5 Lens Model of Ring Optical Resonator ............................ 34
4.6 Small Arm Beam Waist .......................................... 35
4.7 Long Arm Beam Waist .......................................... 37
5.1 Experimental Ring Cavity Configuration ........................ 41
5.2 Periodically Poled KTP .......................................... 42
5.3 Transmission Modes ............................................. 45
5.4 Single Transmission Mode ....................................... 46
5.5 Second Harmonic Generation Bandwith .......................... 47
5.6 Second Harmonic Power .......................................... 48
5.7 Conversion Efficiency ........................................... 49
5.8 Feedback Loop .................................................. 50
5.9 Ring Cavity Error Signal ....................................... 51
5.10 Ring Cavity Servo-lock Circuit ................................. 53
5.11 Schematic of Saturation Spectroscopy in Sr Discharge Cell .... 55
5.12 422 nm Frequency Reference Error Signal ...................... 56
5.13 422 nm Frequency Stabilization Servo-lock Circuit ........... 57
5.14 422 nm System Lay-out ........................................ 58
6.1 1092 nm Laser Absorption Profile .............................. 60
6.2 1092 nm Laser Feedback Signal ................................ 61
6.3 1092 nm Laser Lock Circuit .................................... 62
6.4 Optical Pumping Experiment Illustration ........................ 63
List of Tables
Chapter 1

Introduction

This thesis presents a study of ion dynamics and illustrates progress made toward achieving laser cooling of ultracold neutral plasmas (UNP). Prior to the experiments presented in this document, experiments of this type have never been attempted before with ultracold neutral plasmas. The ultracold neutral plasmas created in our laboratory stretch the boundaries of traditional plasma physics. In our plasmas the electron kinetic energy is in the 1-1000 K range, and the ion kinetic energy is approximately 1 K. These plasmas provide a laboratory realization of a strongly coupled system, where the Coulomb coupling parameter \( \Gamma = \frac{e^2}{4\pi\varepsilon_0 ak_b T} \), which is the ratio of electrostatic potential energy \( \frac{e^2}{4\pi\varepsilon_0 a} \), to thermal energy \( k_b T \), is larger than unity (\( a \) is the inter-particle spacing). When this condition occurs, the electrical interaction energy between charge particles exceed the average kinetic energy, and interesting effects such as frozen ions arranging themselves into lattice structures can potentially occur.

These strongly coupled systems can help reveal the physics of other fields. For example, the Rydberg atom to plasma transition that occur in these systems can
be an analog of the Mott insulator-conductor transition in condense matter physics. Also, the electron ion recombining processes in the plasma could help further the understanding of the production of cold antihydrogen, which is formed by trapped positrons and antiprotons recombining in a plasma.

However, obtaining a large $\Gamma$ to study those systems is prevented by natural mechanisms which leave the system lying on the interface of the weakly and strongly coupled regime. These mechanisms are correlation heating, electron-ion screening, plasma expansion, and velocity changing collisions. A thorough understanding of these processes is crucial if many-body phenomena and phase transitions such as the formation of Wigner Crystal, is to be ever realized in ultracold neutral plasmas.

The study of ultracold neutral plasmas began at the National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland in 1999 [1]. Since then several groups have worked on the theory and conducted experiments using UNPs created from a variety of atomic species. The attraction to these systems ultimately rest in controlling the initial plasma conditions; for example, the plasma density and electron temperature which can be altered easily and with high precision.

There has been many studies on ultracold neutral plasmas. For example, charge particle diagnostic methods use electron dynamics to detect plasma oscillations to reveal plasma expansion dynamics [2] and study electron-ion recombination into Rydberg atoms [3]. In these experiments the authors were able use the plasma oscillations to map the density distribution and reveal the particle dynamics and energy flow dur-
ing the expansion of the plasma. They proved the hydrodynamic model describes the expansion well. In addition, the recombination studies showed that up to 20 percent of the ions recombine with the electrons on a 100 us time scale, which influences the expansion velocity.

Studies of the plasma were also made by monitoring the the ion dynamics in the system. For example, in [4] the density profile of a calcium plasma was obtained using fluorescence imaging. From that it was concluded that the calcium UNP was in the strongly coupled regime. The authors in [5] used absorption imaging to reveal disorder induce heating of the ions. Furthermore, the experiments performed in [6] illustrated the first observation of kinetic energy oscillations of ions in their local potential well, which gives insight on ion-ion collisions in a strongly couple plasma.

Theoretical simulations have been done on plasma as well. Kuzmin et al. in [7] simulated the electron temperature evolution of the ultracold plasma. These results indicated electrons were not correlated due to a host of heating effects in the system. In the article [8], the effect of laser cooling on a ultracold neutral plasma was numerically model. This study revealed that laser cooled ions may exhibit long range ordering resulting in the formation of concentric ion shells. This suggests that a laser cooled UNP can be used to study the physics of liquid to solid phase transitions in plasmas.

This thesis focuses on the use of optical imaging techniques to study early time ion dynamics and the mechanisms that effect laser cooling in an ultracold neutral
plasma. The remainder of this chapter discusses the creation of a strontium UNP. Chapter 2 discusses the ion dynamics occurring in an UNP. Chapter 3 discusses imaging and spectroscopy techniques used to study the ultracold neutral plasma. Chapter 4 and 5 describes the design and construction of the 422 nm laser system used for imaging. Chapter 6 focuses on the 1092 nm laser system and optical pumping experiments. Chapter 7 illustrates progress made towards laser cooling an ultracold neutral plasma. Chapter 8 focuses on determining the collision rates in an UNP by simulating optical pumping data. Chapter 9 is the conclusion of my thesis, which discusses the significance or our results.

1.1 Creation of an Ultracold Neutral Strontium Plasma

The production of the ultracold neutral plasma begins with laser cooled and trapped strontium atoms in a Magneto-Optical Trap (MOT). A MOT is a combination of six laser beams (two for each orthogonal direction) with magnetic fields to restrict the momentum and spatial distribution of neutral atoms in a vacuum. The strontium cooling line is the \(^1S_0\) to \(^1P_1\) transition at 461 nm. The linewidth of the transition is 32 MHz and the saturation intensity, \(I_{\text{sat}}=45 \text{ mW cm}^{-2}\). The 461 nm light was created by frequency doubling light at 922 nm via second harmonic generation. The neutral atom cloud has a temperature of about 20 mK, and its density distribution is Gaussian \(n(r)=n_0 \exp\left(\frac{r^2}{2\sigma^2}\right)\). The peak density at the center of the cloud, \(n_0\), is approximately \(10^{11} \text{ cm}^{-3}\), and the cloud rms width, \(\sigma\), is typically 1 mm. The details
of the MOT are described in [9].

The MOT magnets and lasers are then turned off, and atoms that are excited to the $^1P_1$ level by the MOT lasers are ionized with photons from a 10 ns pulsed dye laser. The pulse laser wavelength is tuned just above the ionization threshold, 412.777 nm. The initial electron kinetic energy ($E_e$) approximately equals the difference between the photon energy and the ionization potential, which is due to the small electron-to-ion mass ratio. The electron temperature $\frac{E_e}{k_b}$ can be as low as the bandwidth of the ionizing laser, which is $\approx 100$ mK with standard pulse dye lasers. For experiments presented in this thesis $\frac{E_e}{k_b}$ is between 10 and 100 K. The initial kinetic energy for the singly charge ions are close to that of the original neutral atoms, in the millikelvin range. The plasma density and electron temperature, $T_e$, is depends on the number of atoms photoionized in the MOT and the wavelength of the photoionizing laser.

The photoionization process to create a strontium UNP uses a pulse dye laser to excite atoms in the $^1P_1$ level to the continuum. In a steady state MOT, the fraction of atoms in the excited state can approach 50%, according to the following equation

$$\rho_{ee} = \frac{cS_0/2}{1 + cS_0 + (2\delta/\gamma)^2},$$  

where $\delta$ is the MOT cooling laser detuning from the atomic resonance, $\gamma$ is the full linewidth at half maximum for the transition, and $S_0 = I/I_{sat}$ is the saturation parameter. The term $c$ is a parameter related to the number of beams, and $I$ and $I_{sat}$ are the MOT laser beam and saturation intensities of the cooling transition.
respectively.

The probability of ionizing the excited state fraction is given by \(1 - \exp\left(\frac{-F}{\sigma_{PI}}\right)\), where \(\sigma_{PI}\) is the cross section for absorption of ionizing photons. The number of photons per unit area in the pulse is, \(F = \int_{\text{pulse}} dt / h\nu_{\text{laser}}\) for instantaneous laser intensity \(I\). Thus with large laser intensities it possible to ionize nearly 100% of the atoms in the intermediate state. For experiments presented in this thesis, a high intensity optical pumping beam at 461 nm is used to excite \(\sim 50\%\) of the atoms to the excited state just before the pulse laser ionizes them.

For a given \(E_e\), there is a threshold number for creating an ultracold neutral plasma. The condition for a sample of ionized gas to be a plasma requires the Debye screening length \(\lambda_D = \sqrt{\frac{e^2 k_B T_e}{e^2 n}}\) to be less than the sample size \(\sigma (\lambda_D < \sigma)\), which is the condition for electron trapping. Electron trapping occurs in a photoionized sample, as a result of an imbalance in the local charge distribution due to the mobility of the ions as compared to the electrons on a short time scale. The resulting charge imbalance creates a Coulomb potential energy well which traps all but a small fraction (< 5%) of the electrons. Simulations show that electrons escape mostly from the edges of the spatial distribution, and the center of the cloud is well describe as a neutral plasma.
Chapter 2

Plasma Dynamics

This chapter gives a description of the phenomena occurring during the evolution of an ultracold neutral strontium plasma. The dynamics can be divided into three phases: disorder induced heating (DIH) of the ions, electron screening of the ion-ion interaction, kinetic energy oscillations, and expansion. As far as the electrons are concerned, they establish thermal equilibrium on a time scale much faster than the ions, which allows us to treat them adiabatically. The details of electron dynamics can be found in the works of P. Gupta [10]. Understanding these dynamics can reveal the physics of the plasma, which is important information for laser cooling.

2.1 Disorder-Induced Heating

Ions in an ultracold neutral plasma are created with very little kinetic energy, since the electrons take away essentially all the excess photon energy during the ionization process due to the small electron-ion mass ratio. However, the ions gain kinetic energy from changes in the Coulomb interaction due to the development of correlations. This causes a significant decrease in the ion coupling parameter, $\Gamma_i$,
This figure is taken from [5]. Ion temperature is determined from the spectral width of the plasma absorption profile. Data shows disorder-induced heating of the ions as the ions equilibrate on a 250 ns time scale.

which leaves the system on the edge of the strongly coupled regime. The increase in kinetic energy due to correlation build up as the ions equilibrate is a process called disorder induced heating. This phenomena was first predicted by Murillo [11] and first experimentally observed by Simien et al. [5]. The increase in kinetic energy corresponds to an increase in temperature as illustrated in Fig. 2.1.

Assuming complete initial disorder, the equilibration temperature of the ions is

$$T_{final} = \frac{2}{3} \frac{e^2}{4\pi\epsilon_0 ak_b} |\hat{U} + \frac{\kappa}{2}|.$$  

(2.1)

The term $\kappa = \frac{a}{\lambda_D}$, with $a = \left(\frac{3}{4\pi n_i}\right)^{-1/3}$ being the Wigner-Seitz radius, describes the
electronic screening. The quantity $\tilde{U}$ is the excess potential energy per ion in units of $\frac{2}{3} \frac{e^2}{4\pi \epsilon_0 a}$ [12]. $\tilde{U}$ complicates Eq. 2.1, since it depends on $T_{final}$ itself, thus an iterative numerical technique must be used to solve for the ion temperature for given conditions, which was done in [6]. The time scale for disorder induced or correlation heating is approximately given by the inverse ion plasma frequency $\omega_{pi}^{-1} = \sqrt{\frac{m_i e^2}{n_i e^2}}$, which represents the amount of time it takes an ion to move an interparticle spacing, $a$, under the influence of a Coulombic force.

Figure 2.2(a) shows the evolution of the ion temperature for three different densities. The data shows the equilibration time to be faster and the temperature hotter for higher densities as suggested by Eq. 2.1, since the value of the plasma oscillation frequency $\omega_{pi}^{-1}$ and $T_{final}$ are density dependent. Figure 2.2(b) shows the data with $T_{i,eff}$ scaled by $T_C = \frac{2}{3} \frac{e^2}{4\pi \epsilon_0 a}$ and time by $\omega_{pi}$. The three curves overlap quite well with slight differences in the temperature axis. Thus, showing $\omega_{pi}$ and $T_C$ are suitable energy and time scale for disorder induce heating.
Figure 2.3: Effective ion temperature versus time after photoionization for various initial electron temperatures [6]. The peak ion density is the same for all the curves.

2.2 Electron Screening

The slight deviation in the temperature axis of Fig. 2.2(b) is due to scaling $T_{i, eff}$ with $T_C$, which does not incorporate the screening of the ion-ion potential by the electrons. For ultracold neutral plasmas, the Yukawa potential $V(r) = \frac{e^2}{4\pi\epsilon_0 r} \exp(-r/\lambda_D)$ is the best model to describe the ion-ion interaction. The use of this model to describe the ion interaction is quantified in Eq. 2.1 by the factor $\kappa = \frac{a}{\lambda_D}$, which depends on electron temperature.

Figure 2.3 is a graph of the effective ion temperature versus time for three different electron temperatures, but same peak ion density $n_i$. This plot illustrates for colder electron temperatures, thus smaller $\lambda_D$, the ion-ion interaction is screened more. The increase in screening reduces the potential energy and therefore the final equilibration temperature.
Figure 2.4: Effective ion temperature for different selected regions of the plasma.

2.3 Kinetic Energy Oscillations

Close inspection of Fig. 2.2(a) reveals that the ion temperature undergoes damped oscillations. These oscillations are due to ions oscillating in their local potential well. Since, during the disorder induced heating phase an ion travels an interparticle spacing and moves to the bottom of its local potential well to a lower potential energy. However, it overshoots this value, and begins to oscillates in the well. This causes an exchange between potential and kinetic energy until the ion energy settles to its equilibrium value as evident in Fig. 2.4. In this figure the effective ion temperature, $T_{i, eff}$, is observed for inner and outer region of the plasma cloud, $r = \sqrt{x^2 + y^2} < .9\sigma$ and $r > 1.48\sigma$. The variation in density causes the ions to oscillate in their local potential well with different frequencies for the respected regions. The area with higher density ($r < .9\sigma$) equilibrates to a higher temperature faster.
Figure 2.5: Absorption Image of an ultracold neutral strontium plasma for several different delay times. Noticeably, the plasma clouds increases in size for increasing time after photoionization.

Kinetic energy oscillations are only evident in the annular regions since there is less density variation. For example, the oscillation period, $\omega_{pi}$, is larger for the inner region as compared to the outer region where the average density is lower. Thus, averaging over the entire cloud would obscure the oscillations (Fig. 2.2(a)), since the motion dephases because of changes in $\omega_{pi}$. The details of annular analysis can be found S. Laha thesis [13]. Also, it is important to note that kinetic energy oscillations have been observed in molecular dynamic simulations of equilibrating strongly coupled plasmas [14], however these were the first experimental observations.
2.4 Plasma Expansion

Figure 2.5 demonstrates absorption images of an ultracold neutral plasma taken for several different delay times after photoionization. Noticeably, the plasma cloud increases in size with increasing time after photoionization. This increase in size is due to the plasma cloud expanding into the surrounding vacuum. The UNP expansion can be attributed to the thermal pressure of the electrons on the ions. This can be understood by recalling that the electrons in the plasma are trapped in the potential well created by the ions as a result of a slight charge imbalance in the system. Therefore, as the electrons move about in this well an effective pressure is exerted on the ions radially outward. This pressure causes the plasma to grow as illustrated in the above figure.

In a regime of no collisional effects, such as three-body recombination and electron-ion thermalization, the plasma expansion can be described very well by the Vlasov equation. This equation governs the evolution of the electron and ion particle distribution functions $f_\alpha(\vec{r}, \vec{v})$ and is given by [15],

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}_\alpha \frac{\partial f_\alpha}{\partial \vec{r}_\alpha} - \frac{q_\alpha}{m_\alpha} \frac{\partial f_\alpha}{\partial \vec{v}_\alpha} \frac{\partial \phi_\alpha(\vec{r}_\alpha)}{\partial \vec{r}_\alpha} = 0.$$  \hspace{1cm} (2.2)

Here $\alpha = e, i$ for electrons and ions respectively, and $m_\alpha$ and $q_\alpha$ represents the electron or ion mass and charge. The term $\phi_\alpha(\vec{r}_\alpha)$ is the total mean-field potential of the system.
In general, Eq. 2.2 does not have an analytic solution, however for our quasi-neutral ultracold plasma having a spherical Guassian distribution function defined as [16],

\[ f_{\alpha}(\vec{r}, \vec{v}) \propto \exp \left[ -\frac{\vec{r}^2}{2\sigma^2} - \frac{m_{\alpha}(\vec{v} - \vec{u})^2}{2k_bT_{\alpha}} \right], \tag{2.3} \]

where \( u(\vec{r}, t) = \gamma(t)\vec{r} \) is the ion expansion velocity; the Vlasov equation has an analytic solution. By substituting Eq. 2.3 into Eq. 2.2, a set of differential equations is obtained that describes the evolution of the plasma size, electron temperature, and expansion velocity in time. The solution to these equations are listed below:

\[
\begin{align*}
\gamma(t) &= \frac{t}{\tau_{exp}^2}, \\
\sigma^2(t) &= \sigma^2(0) \left(1 + \frac{t^2}{\tau_{exp}^2}\right), \\
T_{\alpha}(t) &= \frac{T_{\alpha}(0)}{1 + \frac{t^2}{\tau_{exp}^2}},
\end{align*}
\tag{2.4}
\]

where the characteristic plasma time \( \tau_{exp} \) is defined as

\[
\tau_{exp} = \sqrt{\frac{m_i\sigma(0)^2}{k_b[T_e(0) + T_i(0)]}}. \tag{2.5}
\]

The equations in 2.4 illustrates how initial size and electron temperature determines the plasma expansion. For example, the expression for \( \tau_{exp} \) suggest for large \( \sigma(0) \) the expansion will be slow, and the change in electron temperature is small. Having the ability to predict and control the expansion allows us to create a plasma
that is ideal for laser cooling.
Chapter 3

Imaging and Spectroscopy of an Ultracold Neutral Plasma

3.1 Other Plasma Diagnostic Methods

Various methods have been used to investigate the dynamics of plasmas. For example, experimentalists have applied RF frequencies to resonantly excite electrons. These electrons were monitored using a charge particle set-up to map the density distribution of the plasma. Other studies have used magnetic probes to obtain information regarding the electric and magnetic fields inside a plasma.

These diagnostic methods revealed valuable information about plasmas, however they have several drawbacks that would prevent us from studying the physics we want to learn about from our system. For example, the time resolution of the charge particle detection technique is limited to microseconds due to the time of flight to the detector. This limited resolution would have prevented us from studying equilibration of the ions as described in the previous chapter.
Figure 3.1: Ion Energy Level. The strontium ion has a strong transition in the visible at 421.7 nm. The ions can be optically pumped in the D state, however the imaging laser intensity for experiments in this thesis is well below the $^2S_{1/2} - ^2P_{1/2}$ transition saturation intensity, and is turned on for a few $\mu$s.

3.2 Absorption Imaging of an Ultracold Neutral Plasma

Optical imaging offers a better way to study the ion dynamics in an ultracold neutral plasma. Probing the plasma in this manner via absorption or fluorescence imaging allows for excellent temporal, spatial, and spectral resolution. We can optically image our plasma because strontium ions have an allowed transition in the visible at 421.7 nm (see Fig. 3.1). There are no commercial lasers available at this wavelength, so this light is created by frequency doubling light from an infrared extended cavity diode laser. The design and construction of this purple laser will be discussed in detail in chapter 4 and 5 of this thesis.

All the experiments described in this thesis use optical imaging to study the plasma. To obtain a absorption image of the plasma, a collimated laser beam, tuned near resonance with the principle strontium ion transition illuminates the plasma.
Figure 3.2: Schematic for Absorption Image. This is an illustration of the absorption imaging experimental layout. Near resonant light illuminates the plasma cloud and a shadow is cast on to the CCD camera.

and falls on an image intensified CCD camera. As the beam is absorbed, its intensity decays exponentially through the plasma, and a shadow is cast on the camera. Figure 3.2 is an illustration of the absorption imaging technique. Figure 3.3 shows a typical absorption image.

The decay in the image beam intensity is governed by Beer’s Law:

$$I = I_o e^{-OD},$$

(3.1)

where $OD$ is the optical depth, which represents the column density of atoms along the imaging beam propagation direction. The measured optical depth is defined in terms of the image beam intensity without ($I_{background}$) and with ($I_{plasma}$) the plasma present,

$$OD_{measured}(\nu, x, y) = \ln\left[\frac{I_{background}(x, y)}{I_{plasma}(x, y)}\right].$$

(3.2)
To obtain quantitative information from the images it is useful to define the optical depth theoretically in terms of underlying physical parameters as [17]:

$$OD_{\text{theory}}(\nu, x, y) = \int dz n_i(\vec{r}) \alpha[\nu, T_i(\vec{r}), u^k(\vec{r})], \quad (3.3)$$

where $n_i(\vec{r})$ is the ion density and $\alpha[\nu, T_i(\vec{r}), u^k(\vec{r})]$ is the absorption cross section at the image beam frequency. The absorption cross section, $\alpha$, is a function of ion temperature $T_i(\vec{r})$ due to Doppler broadening, which varies with position according to Eq. 2.1. It also depends on the Doppler shift at $\vec{r}$ due to the expansion velocity $u^k(\vec{r})$, where $k$ refers to the component of $u(\vec{r})$ along the image beam direction. The
absorption cross section for this case is given by a Voigt profile:

$$\alpha(\nu, T_i(\vec{r}), u^k(\vec{r})) = \int ds 3^* \lambda^2 \frac{1}{2\pi} \frac{1}{1 + 4(\frac{\nu - s}{\gamma_{eff}/2\pi})^2} \sqrt{2\pi} \sigma_D(T_i(\vec{r})) \exp \left[ -\frac{(s - (\nu_0 + u^k(\vec{r})))^2}{2\sigma_D(T_i(\vec{r}))^2} \right],$$

(3.4)

where $\sigma_D(T_i(\vec{r})) = \sqrt{k_b T_i(\vec{r})/m_i/\lambda}$ is the Doppler width, $\gamma_{eff} = \gamma_0 + \gamma_{laser}$ is the effective Lorentzian linewidth due to the natural linewidth of the transition, $\gamma_0 = 2\pi \times 20$ MHz, and the laser linewidth, $\gamma_{laser} = 2\pi \times 8$ MHz. The center frequency of the transition is $\nu_0 = c/\lambda$, where $\lambda = 421.7$ nm. The 'three-star' symbol, $3^* = 1$, is a numerical factor that accounts for the polarization state of the ions and imaging laser [18].

### 3.3 Absorption Spectrum of an Ultracold Neutral Plasma

Experimentally the absorption spectrum is obtained by summing the experimental OD over the x and y camera pixel coordinates for the images multiplied by the pixel area taken at different image beam frequencies. Theoretically it is calculated by integrating the optical depth over the x and y coordinates [6]:

$$S(\nu) = \int dxdy OD(\nu, x, y) = \int d^3r n_i(\vec{r}) \alpha[\nu, T_i(\vec{r}), u^k(\vec{r})]$$

(3.5)

Figure 3.4 shows a typical absorption spectrum of the ions and fit. The dominant contribution to the linewidth of the spectrum beyond the natural width is due to Doppler broadening. This makes the spectrum a very accurate probe of the ion velocity profile. From a fit of the experimental absorption spectrum with Eq. 3.5 we
extract the Doppler width, $\sigma_D(T_i(\vec{r}))$. Using this width, the ion temperature $T_i(\vec{r})$ is determined, which can be measured for different times after plasma creation.

In practice, obtaining ion thermal temperature at later times from the spectrum is not possible due to an inability to separate effects of the ion thermal motion and expansion. As a result, the temperatures obtained from fits using Eq. 3.5 are a measure of the ion kinetic energy.
3.4 Fluorescence Imaging and Spectroscopy of an Ultracold Neutral Plasma

The system can also be studied via fluorescence imaging by collecting optical emission from the plasma through a lens system onto a image-intensified CCD camera. The analytical techniques developed to study plasma dynamics via fluorescence imaging and spectroscopy in this thesis were done by S. Laha and details can be found in his thesis [13].

Figure 3.5 is a diagram of the fluorescence image system used in our lab. A near
resonance laser beam propagates along the x direction and illuminates the plasma. The fluorescence is collected in the perpendicular z direction onto the CCD camera. Similar to absorption methods, the fluorescence can be related to underlying physical parameters and is given by,

\[ F(\nu, x, y) \propto \int ds \frac{\gamma_0}{\gamma_{\text{eff}}} \times \frac{d\nu T_i(\vec{r})}{2\pi \sigma D(\vec{r})} \exp \left[ -\frac{\left( s - (\nu_0 + u^k(\vec{r})) \right)^2}{2\sigma D(\vec{r})^2} \right]. \tag{3.6} \]

Fluorescence images can be analyzed in different ways. For example, to obtain information regarding the density distribution of the plasma cloud a series of images taken at equally spaced frequency is summed over the entire ion resonance and fitted to the integral of \( F(\nu, x, y) \) over frequency, which is given by the following equation

\[ \int S(d\nu, x, y) \propto \int dzn_i(\vec{r}) = \sqrt{2\pi} \sigma n_0 \exp \left[ -\frac{(x^2 + y^2)}{2\sigma^2} \right], \tag{3.7} \]

where the expression of the right-hand side of Eq. 3.7 is the areal plasma density, \( n_{\text{areal}} \). The quantities \( n_0 \) and \( \sigma \) are the peak plasma density and rms width of the cloud respectively. It is important to note that Eq. 3.6 is valid only if the plasma is much smaller than the image beam size.

Fluorescence spectrum of the plasma is obtained by integrating \( F(\nu, x, y) \) over
some region in the plasma. The resulting expression is the following equation

$$\int_{reg} dxdy F(\nu, x, y) \approx \frac{N_{i,reg}}{\sqrt{2\pi} \sigma_D(T_{i,reg})} \int ds \frac{\gamma_0/\gamma_{eff}}{1 + 4(\nu - s)^2 / \gamma_{eff}^2} \exp \left[ \frac{[s - (\nu_0 + u_{reg})]^2}{2\sigma_D(T_{i,reg})^2} \right].$$

(3.8)

Here $N_{i,reg}$, $T_{i,reg}$, and $u_{reg}$ are the number of ions, average ion temperature, and expansion-induced Doppler shift in a particular region of interests. Fluorescence spectroscopy has a powerful advantage over absorption in that it can be obtained in different regions. As a result, by evaluating the center of the plasma, where the expansion is zero, the thermal temperature of the ions is measured. In chapter 7 this technique is used to look for changes in the thermal temperature of the ion cloud, which are signs of laser cooling.
Chapter 4

Second Harmonic Generation in an Optical Resonator

This chapter describes the construction of a purple laser at 421.7 nm to optically study and laser cool an ultracold strontium neutral plasma. This wavelength is generated by frequency doubling an existing infrared commercial laser inside an external optical resonator. In this chapter the theoretical concepts used to design the 421.7 nm laser are discussed.

4.1 Second Harmonic Generation

Non-linear optical phenomena in the interaction of light with a particular media is a result of the non-linear nature of the polarization, which can be written in terms of the electric field E as:

\[ P = \varepsilon_0 \chi_1 E + \varepsilon_0 \chi_2 E^2 + \varepsilon_0 \chi_3 E^3, \]

(4.1)
where $\chi_1$ is the linear susceptibility, $\chi_2$ is the second order susceptibility, and $\chi_3$ is the third order susceptibility. The term $\chi_2$ is responsible for second harmonic generation.

Second harmonic generation is a non-linear process in which an electromagnetic wave with frequency $\omega$ is converted into one at frequency $2\omega$. For example, consider an electromagnetic field with frequency $\omega_1=\omega$ traveling along the z-axis through a crystal with a non-zero $\chi_2$. This interaction of light with the material will create a polarization wave in the crystal with frequency $\omega_2=2\omega$. This polarization wave will then produce radiation at $\omega_2$. The power of this radiation is related to the power at $\omega$ with beam area $A$ by the following relation:

$$P_{\omega_2} = \left[ \frac{2n_o^3 \omega_2^2 d_{eff}^2 L^2}{A} \right] P_{\omega_1}(\frac{\sin \frac{\Delta k L}{2}}{\Delta k L})^2 = \xi_{nL} P_{\omega_1}^2 (\frac{\sin \frac{\Delta k L}{2}}{\Delta k L})^2,$$

where

$$\Delta k = \frac{2\omega_1(n_1 - n_2)}{c}.$$  \hfill (4.3)

$L$ is the length of the medium, $d_{eff}$ is the nonlinear coefficient of the doubling crystal, and $n_o = 377/n_1$ [19]. The term $\xi_{nL}$ is the nonlinear conversion efficiency. $n_1$ and $n_2$ are the index of refraction at $\omega_1$ and $\omega_2$ respectively.

### 4.2 Phase Matching

In Eq. 4.2 we can see that second harmonic power is maximized when $\Delta k L=0$. When this happens, the second harmonic wave and fundamental wave inside of the
Traditionally, phase matching can be achieved either by changing the angle at which the fundamental wave propagates with respect to the optical axis of the crystal (a), or by tuning the temperature of the crystal (Type I) (b), since the index of refraction is also temperature dependent.

Figure 4.1: Traditionally, phase matching can be achieved either by changing the angle at which the fundamental wave propagates with respect to the optical axis of the crystal (a), or by tuning the temperature of the crystal (Type I) (b), since the index of refraction is also temperature dependent.

Material are phase matched. Physically this occurs when \( n_{\omega_1} = n_{\omega_2} \), which means that both waves must have the same phase velocities inside the crystal. If the phase velocities of the two waves are not equivalent, then second harmonic waves generated at different planes throughout the crystal will destructively interfere with each other, as described by the sinc function in Eq. 4.2, thereby resulting in low \( \omega_1 \) to \( \omega_2 \) conversion efficiency.

Usually in materials, \( n_{\omega_1} > n_{\omega_2} \), therefore phase-matching is not achievable (dispersion-effect). However, in birefringent materials, materials that possess different values of indices of refraction in different directions, the phase matching condition \( n_{\omega_1} = n_{\omega_2} \) can be satisfied: light with frequency \( \omega_1 \) is polarized along one axis of the crystal, while light with frequency \( \omega_2 \) is generated along another perpendicular crystal axis [20].

Traditionally, phase matching can be achieved either by changing the angle at
which the fundamental wave propagates with respect to the optical axis of the crystal, or by tuning the temperature of the crystal, since the index of refraction is also temperature dependent (see Figure 4.1). This has a major drawback, since in both cases, there is non-collinear propagation of the fundamental and second harmonic wave to enable growth of the harmonic wave along the material. This restricts access to the highest nonlinear coefficients, $d_{eff}$, therefore limiting the doubling efficiency.

Recently, techniques have been developed to enable continuous growth of the harmonic wave along the device, called Quasi-Phase Matching (QMP)[21]. QMP involves repeated inversion of the relative phase between the interacting waves, such that the waves still propagate at different phase velocities, but an accumulated phase mismatch is prevented.

Figure 4.3 is an illustration of the output second harmonic power from a nonlinear crystal for three various phase matching scenarios: perfectly phase matched, quasi-phase matched, and non-phase matched. For perfectly phase matched second harmonic generation, after the interacting waves travel a coherence length, $L_c = \frac{\lambda}{4|n_{2\omega}-n_\omega|}$,
Figure 4.3: This figure is an illustration of the growth of second harmonic intensity for perfectly phase matched, quasi-phase matched, and non-phase matched scenarios.

the harmonic power increases quadratically. For the non-phase matched case, the harmonic power decreases to zero as expected. For the quasi-phase matched situation, the relative phase is inverted every coherence length such that on average the harmonic power grows.

One way to invert the phase is by periodically alternating the index of refraction or equivalently the nonlinear coefficient (Fig. 4.2). This can be achieved by periodically poling a crystal. A material is periodically poled by applying electric fields to reverse its domains in a localized region [22]. The crystal used to produce purple light for experiments in this thesis is Periodically Poled $KTiOPO_4$ (ppKTP). The advantage of ppKTP is that it is a thicker crystal and therefore can be poled at much lower electric fields, which limits both domain spreading and refractive damage of the
Figure 4.4: Our enhancement cavity is a ring resonator consisting of two flat mirrors, one being designated as the input coupler, and two curved mirrors with radius $R_1=R_2=50$ mm. A beam with waist $s_1$ passes through a coupling lens of selected focal length and enters into the optical cavity. The parameter $s_2$ and $s_3$ are the waists in the cavity’s long and short arms. The tight focus is located between the two curved mirrors.

crystal. In addition, it is optically transparent at both the fundamental and second harmonic wavelengths, and both interacting waves can have the same polarization making higher $d_{eff}$ accessible.

4.3 Enhancement Cavity

Equation 4.2 illustrates that the second harmonic power depends quadratically on the fundamental power. Thus, large amounts of fundamental power will result in high conversion efficiencies. Unfortunately, standard inexpensive continuous wave lasers do not produce the high powers needed that will yield significant conversion from infrared-to-purple. However, we can enhance the modest powers from a commercial laser with the use of an optical resonator.

An optical resonator is a set of two or more mirrors configured to allow light to propagate in a closed path. The enhancement of an optical resonator results from the
effective number of round-trips the light makes along its closed path. For an optical ring resonator the circulating power inside the cavity is expressed as [18]:

\[ P_c = P_{\text{input}} \frac{T_1}{[1 - \sqrt{(1 - T_1)(1 - \epsilon)(1 - \gamma P_c)}]} = bP_{\text{input}}, \quad (4.4) \]

where \( P_{\text{input}} \) is the input power of the laser, \( T_1 \) is the transmission factor of the input coupler, \( \epsilon \) is the resonator round-trip parasitic loss excluding the input mirror transmission \( T_1 \) and conversion to blue loss, and \( b \) is the effective number of photon round-trips in the cavity. The term \( \gamma \) is defined as

\[ \gamma = \gamma_{\text{eff}} + \gamma_{\text{abs}}, \quad (4.5) \]

where \( \gamma_{\text{eff}} \) is the efficiency for the infrared-to-purple conversion loss, \( P_{2\omega} = \gamma_{\text{eff}} P_c^2 \).

This describes the fraction of infrared light loss per pass to second harmonic generation, and similarly \( \gamma_{\text{abs}} \) is the efficiency of the second harmonic light being absorbed, \( P_{2\omega} = \gamma_{\text{abs}} P_c^2 \).

Figure 4.4 illustrates our set-up of an optical resonator. The design is a symmetric bow-tie configuration, which consists of two flat mirrors, one being designated as the input coupler, and two curved mirrors with radius-of-curvatures \( R_1 \) and \( R_2 \) (we specialize \( R_1 = R_2 = 50 \text{ mm} \)) separated by a distance twice its focal length. Light from an external laser enters the cavity through the input coupler-mirror, which is ideally designed to be partially reflecting such that its transmission loss is equal to the sum of
all other losses in the system. The lens, shown in Fig. 4.4 is used to match the beam waist $s_1$ from the external laser source into that of the optical resonator. The terms $s_2$ and $s_3$ are the minimal beam waist in the long and short arms of the resonator.

4.4 Guassian Modes

Following [23], the electric field component for laser light traveling in the $z$ direction can be written as

$$u = \psi(x, y, z) \exp(-jkz)$$  \hspace{1cm} (4.6)

where $\psi$ is the transverse electric field pattern of the laser beam. The wave equation in cylindrical coordinates that describes these modes is the following:

$$\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial \psi}{\partial r} - j2k_c \frac{\partial \psi}{\partial z} = 0,$$  \hspace{1cm} (4.7)

where $k_c$ is the vacuum wave vector. There are many solutions to the above equation having different transverse modes (spatial patterns). The lowest order transverse mode of Eq. 4.7 is called the $TEM_{00}$ or Gaussian mode and is ubiquitous in laser systems used for atomic physics research. This mode is circular in its transverse dimension, and has very nice focusing properties. Mathematically, it is expressed as

$$\psi = \exp[-j(P(z) + \frac{kr^2}{2q(z)})],$$  \hspace{1cm} (4.8)
where $q(z)$ is the confocal parameter, describing the variation in beam intensity with distance from the optical axis, and $P(z)$ is the complex phase shift. These two parameters are defined as the following:

$$\frac{1}{q(z)} = \frac{1}{R(z)} - \frac{j\lambda_0}{\pi s(z)^2},$$  

(4.9)

$$P(z) = (kz - \Phi),$$  

(4.10)

where $s(z) = \sqrt{s_0^2[1 + (\frac{\lambda z}{\pi s_0^2})^2]}$ is the $1/e^2$ intensity radius or spot-size of the gaussian beam, $R(z) = z[1 + (\frac{\pi s_0^2}{\lambda z})^2]$ is the wavefront radius of curvature, and $\Phi(z) = \arctan(\frac{\lambda z}{\pi s_0^2})$. The quantity $s_0$ in the expression for $s(z)$, $R(z)$, and $\Phi(z)$ is the beam waist. Following Eqs. 4.6 - 4.10 the gaussian beam transverse intensity pattern is written as:

$$I(x, y, z) = \frac{2P}{\pi s(z)^2} \exp\left[-2(x^2 + y^2)\frac{s(z)}{s(z)^2}\right],$$  

(4.11)

where $P$ is the power of the laser beam.

### 4.5 Modeling Gaussian modes in an optical resonator

Enhancement of the input power in an optical resonator of the $TEM_{00}$ mode from an external laser source requires that the mode of the external laser be congruent to the mode emerging from the cavity. Thus, we need to know the beam profile of the external laser source and beam profile of the cavity modes.
Figure 4.5: A lens system equivalent to the optical resonator used in our set-up, which consists of two concave and flat mirrors, and an ppKTP crystal.

For our applications, we determine the beam profile of the lowest-order resonator modes (TEM$_{00}$), since the external laser mode is a TEM$_{00}$. We determine the profile of these modes by modeling the optical resonator as a periodic sequence of lenses (Fig. 4.5). The focal lengths of the lenses in the model are the focal length $f = R/2$, where $R$ is the radius of curvature of the two resonator curved mirrors.

The confocal parameter $q(z)$ of a guassian mode transforms through any optical system according to the ABCD law of ray optics [24]. Particularly, the confocal parameters $q_1(z)$ and $q_2(z)$ before and after an optical element are related by the following equation:

$$q_2(z) = \frac{Aq_1(z) + B}{Cq_1(z) + D},$$

(4.12)

where A, B, C, and D are the entries of the ABCD matrix or ray for the optical
Figure 4.6: This is a plot of $s_3$ versus $d_{2\rightarrow3}$, the distance between the two curved mirrors for $R1=R2=50$ mm, and taken into account crystal optical length. For ppKTP the crystal index of refraction and length are $n_{\text{crystal}}=1.840$ and $d_{\text{crystal}}=10$ mm.

elements. Using this rule, we can determine the beam radius-of-curvature and spot-size inside the optical resonator, by requiring that the confocal parameter $q(z)$ be the same after a round trip in the cavity.

For our model (Fig. 4.5), the ABCD matrices $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ that we need to describe our system are given by:

$$T_{\text{distance}} = \begin{pmatrix} 1 & \frac{d}{n} \\ 0 & 1 \end{pmatrix}, \quad (4.13)$$

and

$$T_{\text{lens}} = \begin{pmatrix} 1 & 0 \\ \frac{-2}{R} & 1 \end{pmatrix}, \quad (4.14)$$

where $T_{\text{distance}}$ is the transfer matrix that describes the transformation over medium of length $d$ and index $n$. $T_{\text{lens}}$ is an ABCD matrix that describes the transfer of a beam through a thin lens of focal length $f$. 

35
The resultant ABCD matrix is found by applying the transfer matrices of Eqs. 4.13 - 4.14 for one round trip in the cavity, starting just after left-hand curve mirror. From this the resultant ABCD matrix is given by

\[
\begin{pmatrix}
A & B \\
C & D \\
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
-\frac{2}{R_1} & 1 \\
\end{pmatrix} \begin{pmatrix}
1 & d_1 + 2d_2 \\
0 & 1 \\
\end{pmatrix} \begin{pmatrix}
1 & 0 \\
-\frac{2}{R_2} & 1 \\
\end{pmatrix} \begin{pmatrix}
1 & d_3 \\
0 & 1 \\
\end{pmatrix}
\] (4.15)

where \(d_3\) is optical path length between \(R_1\) and \(R_2\), taken into account the index of refraction of the ppKTP crystal. By setting \(q_1(z) = q_2(z)\) in Eq. 4.12, and using the ABCD elements of Eq. 4.15 we obtain an expression for the resonator beam size at first curved mirror in the \(d_3\) path,

\[
s^2 = \frac{2\lambda B}{\pi \sqrt{4 - (A + D)^2}}.
\] (4.16)

As a result of symmetry, the minimum beam size is located at the center of the crystal, which is easily determined by propagating the result of Eq. 4.16 to that location. This value, \(s_3\), is important since according to equation 4.2 the second harmonic power depends quadratically on the beam waist in the crystal. Figure 4.6 is a plot of \(s_3\) as a function of \(d_{2-3}\), the optical path length between the curved mirrors \(M_3\) and \(M_4\).

The beam waist and location in the \(d_1\) arm was calculated using a different ap-
**Figure 4.7:** This is a plot of the beam waist in the long arm of our resonator versus $d_{2-3}$, the optical path between the two curved mirrors, for $R_1=R_2=50$ mm. mm.

The approach described in [25], and is the following expression

$$s_2 = s_c \sqrt[4]{\frac{x(1-x)(1+\alpha_2-\alpha_2x)}{(1-\alpha_2x)}}.$$  \hspace{1cm} (4.17)

and its location $z_2$ is

$$z_2 = \frac{d_1}{2}.$$ \hspace{1cm} (4.18)

The term $s_c$ in Eq. 4.17 is defined as $\sqrt{\frac{\lambda R_1}{2\pi}}$. The quantities $x=\frac{d_1}{R_1}$ and $\alpha_2=\frac{2d_2+d_3}{d_4}$. Figure 4.7 is a plot of the beam waist in the $d_1$ arm as a function of the separation between the two curved mirrors, and by Eq. 4.18 it is located at half the distance between the two flat mirrors. This waist is plotted as a function of the distance between the two curve mirrors because the separation in this arm is vital for optimizing second harmonic generation. It is important to note, both methods are valid in calculating the beam waist in each arm, however the equations obtained from the analysis in H.
Abitian et al. are obviously much simpler and straightforward to apply.

4.6 Longitudinal Modes

The build up of a transverse mode in a given optical resonator can only occur at certain frequencies. This discrete set of resonance frequencies are the longitudinal (axial) electromagnetic modes of the optical resonator. The frequency of these modes are determined by the resonance condition that the round-trip phase shift of the resonator mode must be an integer multiple of $2\pi$. This is mathematically expressed as:

$$\frac{4\pi \nu L}{c} - \theta_{nm} = 2\pi p, \tag{4.19}$$

where the term $\frac{4\pi \nu L}{c}$ is the axial mode phase shift, and $p$ is the axial mode integer. The term $\theta_{nm}$ is the transverse mode phase shift having mode integers $n$ and $m$, which varies for different modes. The resonance frequencies obtained from Eq. 4.19 are the following:

$$\nu_{pnm} = p \frac{c}{2L} + \frac{\theta_{nm} c}{4\pi L} \tag{4.20}$$

where $L$ is the cavity length and $c$ is the speed of light in vacuum. In Eq. 4.20, $\frac{c}{2L}$ is the free-spectral range (FSR) of the optical resonator, which is the frequency separation between adjacent $TEM_{00}$ longitudinal modes in Hz. The transverse mode spectrum is described by the term $\frac{\theta_{nm} c}{4\pi L}$ and will be illustrated and briefly discussed in Chapter 5.
4.7 Resonator Losses

The frequency criterion for optical waves to exist inside a resonator is relaxed, when the resonator has losses [26], for example, when the mirrors are not perfect reflectors. The losses of a cavity are describe by the finesse $F$, which is expressed in terms of the overall losses in the system $\alpha$ as:

$$F = \frac{\pi \exp[-\alpha]D}{1 - \exp[-2\alpha]D} \approx \frac{2\pi}{\alpha D}, \quad (4.21)$$

where $\alpha$ is given by

$$\alpha D = \ell + C + \ln \frac{1}{R_a R_b}, \quad (4.22)$$

where $\ell$ is the round-trip parasitic loss and $C$ is the infrared blue conversion loss. The term $\ln \frac{1}{R_a R_b}$ is losses due to mirror reflectivities. For $R_b \approx 1$ Eq. 4.21 reduces to the following:

$$F \approx \frac{2\pi}{\ell + C + T_a}, \quad (4.23)$$

where $T_a \approx 1 - R_a$ is transmission of the input-coupler mirror. In the presence of these the modes are no longer discrete sharp peaks as a function of frequency, but have a spectral full-width-half-maximum (FWHM) $\Gamma$ given by:

$$\Gamma = \frac{FSR}{F}. \quad (4.24)$$
Chapter 5

Experimental Details

This chapter discusses the experimental procedure to create a frequency stabilize 421.7 nm laser to cool the strontium ions.

5.1 Experimental Apparatus

The experimental set-up used to produce light at 422 nm is displayed in Fig. 5.1. In Fig. 5.1, p-polarized light is emitted from a Toptica single frequency high powered tunable diode laser and coupled into a optical fiber. The output light from the fiber ranges in power from 10-120 mW and has a beam waist of 82.9 µm located approximately 12 cm behind the output fiber head.

The light from the fiber passes through a $f = 200$ mm focal length lens (not shown), to transfer its waist from 82.9 µm to a new waist of 231 µm (for mode matching) and Electro-Optical Modulator (EOM), before entering the optical resonator. The optical resonator is a symmetric bow-tie configuration, which consists of a flat input coupler (M1) that transmits 5% at 844 nm, a high reflecting (HR) mirror (M2) mounted on a piezo-electric transducer (PZT), and two HR mirrors (M3 and
Figure 5.1: Schematic of the experimental configuration for frequency doubling a cw infrared laser at 844 nm using ppKTP in a ring resonator.

M4) of 50 mm radius of curvature. The ppKTP crystal is placed between M3 and M4, and M4 serves as an output coupler for the generated purple light.

The ppKTP crystal is 1 mm thick, 10 mm long, and 2 mm high of periodically polled KTP. The periodic poling has a period of 3.94 µm, which is twice the coherence length $L_c$. The poling only extends to 90 % of the crystal physical length, with a 40-60 % duty cycle. The end faces of the crystal are flat, and have anti-reflection coating for the 844 nm and 422 nm light (Fig.5.2).

The ppKTP crystal is wrapped in indium foiled and resides inside an aluminum enclosure (not shown). This enclosure sets on top of a Thorlabs 1-3 amp thermoelectric cooler for active control of the crystal temperature. In turn, the thermoelectric cooler resides on top of a New Focus xyz kinematic mount, which is used for moving the crystal with micro-meter precision. Small adjustments in crystal position
• $d_{\text{eff}} = 17 \text{ pm/V, } x = 2 \text{ mm, } y = 1 \text{ mm, } z = 10 \text{ mm}$
• $n_{2w} = 1.949, n_u = 1.842$
• the crystal is designed for phase matching close to room temperature
• poling with an effective length equal to 90% of physical length and periodic domains with 40-60% duty cycle

Figure 5.2: This is a pictorial illustration of our ppKTP crystal (not to scale). The stripes are an attempt to illustrate that poling does not extend through the entire crystal.

is necessary to identify a spot without any defects in the poling domains to obtain high conversion efficiency.

All the optics of the ring cavity are mounted on an aluminum optics bread board made in the machine shop (see Appendix A). This bread board is covered with fiber glass to protect the optics. In addition, $O_2$ is pumped into the housing to alleviate the long term effects of humidity on the crystal. Light is allowed to enter and leave the housing via tiny ports. The reflected and transmitted IR light from the optical resonator are collected on Thorlabs photo-diodes, and the cavity mode spectrum is displayed on an oscilloscope.
5.2 Mode Matching

Since the fundamental beam parameters of the cavity and external laser source is known, the gaussian modes of the laser source and optical resonator can be matched. If the modes of the two systems are not matched, the $TEM_{00}$ mode output of the external laser source will couple into several transverse modes of the cavity, which will limit the enhancement of the external laser’s fundamental mode. Thus, to prevent the excitement of additional resonator modes, a lens is used to transform the $TEM_{00}$ from the laser into that of the cavity. This phenomena is known as mode matching.

In general, to mode match any two systems, only the beam waists and respective locations are needed to be known for both systems. Once that information is at hand, the problem is to determine the focal length and the distances of the lens with respect to each waists that are needed to match the two systems, as illustrated in Fig. 4.4.

In practice, the input-coupler mirror of this optical resonator is a fused silica substrate. This substrate acts as a plano-concave lens of focal length $-2R$. This effect, in turn, could causes the waist and its location of the beam that would emerge from the cavity to be different from that of the resonator. Therefore, in order to properly mode match we must take into consideration this lens action, and determine the size and location of this virtual beam [24]. However, for our cavity the input-coupler mirror is flat, thus the waist of a virtual beam emerging from the cavity is the waist in the cavity $d_1$ arm, $s_2$.

The lens needed to mode match the beams of the laser-resonator system, must be
a focal length $f$ that is greater than the characteristic mode matching length given by [23]:

$$f_o = \frac{\pi s_1 s_2}{\lambda}, \quad (5.1)$$

where $s_1$ is the input laser beam waist and $s_2$ is the cavity waist in the long arm.

The distances of the waists for both systems from the mode matching lens were determined using the following equations:

$$d_1 = f \pm \frac{s_1}{s_2} \sqrt{f^2 - f_o^2}, \quad (5.2)$$

and

$$d_2 = f \pm \frac{s_3}{s_1} \sqrt{f^2 - f_o^2}. \quad (5.3)$$

For our system the optical path difference between the two curved mirrors is $d_{\text{short}}=0.053$ m, the distance between the two flat mirrors is $d_{\text{long}}=0.1989$ m, and the vertical separation between short and long arm paths is $d_{\text{sep}}=0.03175$ m. These values correspond to a long arm waist of $s_2=257$ µm, and is located $z_2=d_{\text{long}}/2$ from the input coupler mirror. Using these values, along with the beam waist of the external laser and the above equations, we determined our mode matching distances.

### 5.3 Cavity Modes

In our experimental configuration we use our optical resonator as a scanning interferometer in order to monitor the mode spectra. As we scan the length of our
cavity, when the laser sequentially comes into resonance with a cavity longitudinal mode, light enters the cavity, and excites the cavity modes. Mathematically, the laser-cavity resonance condition for two adjacent modes is expressed as:

\[ f_l = \frac{pc}{2L} = \frac{(p+1)c}{2(L + \Delta L)} \]  \hspace{1cm} (5.4)

where \( f_l \) is the laser frequency, \( p \) is the axial mode number, and \( \Delta L = \alpha \Delta V \) is the change in cavity length (corresponding to a FSR) in terms of a change in PZT voltage \( \Delta V \). The term \( \alpha \) is the PZT voltage to length proportionality constant.

Figure 5.3 is the mode spectrum of our optical resonator. The large sharp peaks, \( p \) and \( (p+1) \), are the \( TEM_{00} \) of the cavity, which are separated by one FSR \( (\Delta V \approx 100V) \).
Figure 5.4: This is a plot of a single transmission mode of the 844nm IR enhancement cavity.

The next taller peak is a higher-order transverse mode, which is shifted from the fundamental modes by $\frac{1}{3}$ FSR. From this figure the measured FSR of our optical resonator is 294.7 MHz, which matches well with the value calculated using Eq. 5.4.

Next, we determined the FWHM of our system by examining a single cavity transmission mode. From Fig. 5.4 we determined the FWHM of the transmission mode to be approximately 20 MHz. This implies the finesse, $F$, of our cavity is 14.735 ($F = \frac{FSR}{FWHM}$).
5.4 Phase Matching and Second Harmonic Power

Light at 422 nm is produced by means of temperature-tuned phase matching. At \( \sim 30^\circ C \), we visibly see 422 nm light emitted from the cavity. It is separated from the fundamental light by transmission through (M4) which is coated for high transmission at 422 nm and low transmission at 844 nm.

Figure 5.5 is a plot of the harmonic power (in the locked state) versus temperature for an input infrared power of 90 mW. The PZT for these measurements was controlled such that its length always corresponds to maximal build-up inside the cavity. In this locked state of the cavity, for 90 mW of fundamental power, the maximum blue power occurred at a temperature of 32.04°C. We took the temperature bandwidth for phase
Figure 5.6: Generated second-harmonic power as a function of the mode-matched fundamental power.

matching in our configuration as the FWHM of this peak, which is $\sim 3^\circ C$.

Figure 5.6 is a plot of the harmonic power at 422 nm as a function of the fundamental power. The temperature was varied for each data point to maintain optimal phase matching. For a maximum fundamental power of 90 mW out of our fiber, we get a peak second harmonic power (locked) of 32 mW in a single longitudinal mode. The curve illustrates that the harmonic power grows linearly with input power not quadratically as suggested by Eq. 4.2. This indicates that the system is operating in the low input power regime.

Figure 5.7 is a plot of the optical-to-optical conversion efficiency as a function of incident power. The data illustrates the conversion efficiency has not saturated.
Figure 5.7: Conversion efficiency as a function of mode-matched fundamental power. The asterisks are calculated from the measured harmonic output power.

with fundamental power for our set-up. The peak conversion efficiency value is 35 %, which is much less than the maximum efficiency of 60% obtain in [27]. Again, this is a result of having a 100 mW limitation on input power from the external laser source as compared to experiments done by Goudazi et al. having input powers of 400 mW.

5.5 Enhancement Cavity Feedback Electronics

An optical resonator is prone to external perturbations from the environment. For example, acoustical vibrations from nearby mechanical devices and thermal expansions and contractions due to temperature fluctuations can prevent our blue laser source from having a stable intensity output for cooling experiments. However, by
implementing feedback electronics we can achieve the desired state for our laser system.

Figure 5.8 is a diagram of the feedback network for our optical resonator. The network consists of four key elements: 1) The laser, including the input control of its frequency. This portion changes control voltage into laser frequency. 2) Cavity and RF electronics, which transform the laser frequency into an error signal. 3) Locking Electronics, the heart of which is an integrator, which changes volts of error signal into volts of control. 4) Summing junction, it is used for purposes of feedback into the laser system [28]. Overall, the loop diagram of Fig. 5.8 controls the laser frequency to lock the laser to the peak of the transmission mode, such that build-up is always maximal inside the cavity.

5.6 Error-Signal

In our feedback control network, whether or not the cavity length corresponds to a transmission maximum is indicated by the error signal. For our system, this is a voltage signal that is a function of laser frequency that contains essential information
Figure 5.9: Error Signal for IR enhancement cavity produced by applying RF side-bands on 844 nm light. We scan the cavity in this illustration.

about the location of the modes inside the cavity. We produce this electronic signal to stabilize our system via the Pound-Drever-Hall method [29]. Our experimental set-up for this is also illustrated in Fig. 5.1.

The infrared light from the laser is frequency modulated at 23 MHz by a Electro-Optic-Modulator. This light is steered into the cavity where a fraction of it is reflected and transmitted. The reflected beam, after a series of optical elements, falls on a fast Thorlabs photo-diode. We then perform phase detection at 23 MHz using an electronic mixer from mini-circuits (not shown) to produce our error (demodulated) signal.

Figure 5.9 is a scope-trace of the error signal used to lock the 844 nm laser to the
peak of the transmission mode. The signal can be derived using analysis given in [30]. It shape is essentially the derivative of the reflected mode line shape. In addition, the error signal is antisymmetric, being negative on one side of the mode, positive on another, and zero at the cavity resonance. These features are important, because they indicate to the laser input controller, which direction to respond to compensate for external perturbations. The error signal in our system has some distortion as a result of EOM side-band pick-up in the cavity and modulation of the 844 nm laser current, which will be discussed latter.

We use the error-signal obtained as described above to lock the laser to the peak of our Fabry-Perot cavity transmission, to maintain maximal circulating and 422 nm power for each setting. The linear range about the zero, is called the locking slope (range), and gives a typical tuning range of .4 V per 20 MHz of the cavity mode.

5.7 Electronic Feedback Circuit

The electronic circuit in Fig. 5.10 is used to adjust and feedback the output voltage of the error signal to the cavity such that its length corresponds to a transmission peak of the optical resonator. The heart of this circuit is an integrator, which has an output voltage, $V_{\text{out}}$, given by:

$$V_{\text{out}} = -\frac{1}{RC} \int V_{\text{input}} dt,$$  \hspace{1cm} (5.5)

where R is the input resistor and C the feedback capacitor.
Figure 5.10: Servo-lock Circuit is used to lock the laser to the cavity.

If a perturbation shifts the cavity from resonance, a non zero voltage error signal will be supplied to the integrator. For this input signal, the integrator’s output signal rises steadily (integration over time). The rising output signal is amplified by the output stage of the servo-lock circuit in Fig. 5.10 and is fed back into the cavity driving the PZT (input control), which sweeps the cavity back to the cavity resonance condition. As the cavity length approaches the resonance condition, the error signal reaches zero volt. The integrator now maintains a zero volt output level, until another disturbance occurs.
5.8 Procedure to lock the laser

First, the initial settings are such that the servo-loop in Fig. 5.10 is open, and only a ramp voltage from a function generator is applied to the Ramp Input, which is fed to the PZT to scan the optical resonator. Next, we close the servo loop by switching the servo electronics from scan to lock mode using SW2 so that the error signal is sent to the PZT. The circuit acquires the cavity resonance in this state by allowing the ramp signal to continue to pass through via another current path to the PZT until the cavity transmission signal is peaked. Now, the servo-loop is lock at the peak of the cavity transmission. Finally, we optimize the gain $G$ of the output signal to the PZT control to get a tight lock.

The circuit diagram also includes an additional servo network, to achieve robust cavity to laser locking. The feedback network to the PZT maintains a cavity resonance by displacing a mirror. This method has a response time that is inadequate for maintaining a stable lock in our lab. For a faster response time we included a servo-network to the 844 nm laser current modulation input, because the current source has a higher frequency response (16 kHz). This means the feedback response is very quick (response time is on time-scales proportional to the inverse of the servo-bandwidth). With this ability, we can compensate for external perturbations beyond the bandwidth of our cavity PZT response. As a result, we can maintain a stable lock for hours.
5.9 Frequency Reference using Metrology

The wavelength of the 422 nm laser in our experiment is determined by using a Burleigh WA-100 wavemeter. However, to set the wavelength to a precise value for laser cooling and keep it, the laser must be locked to an atomic reference. Thus, saturation spectroscopy is done in a strontium discharge cell to frequency reference the 422 nm laser.

Figure 5.11 is a schematic of saturation spectroscopy in a strontium discharge cell. Saturation spectroscopy is used to obtain a Doppler free electronic signal to lock the 422 nm laser to the strontium ion transition. In this set-up a weak probe beam and a high intensity pump beam are aligned to counter-propagate and overlap.
Figure 5.12: Electronic Error signal generated from saturation spectroscopy in strontium discharge cell.

each other in the discharge cell. Since, these two beams are propagating in opposite directions only a narrow velocity class of the atoms centered about $v=0$ will interact with both beams. The pump beam saturates this velocity class, so less atoms are in the ground state for the probe beam to absorb. This creates a doppler free peak in the absorption spectra using the probe beam. Since the pump beam is modulated at 20 MHz using an EOM (not shown), a doppler free electronic signal is generated using Pound-Drever Hall technique to lock the laser to the ion transition (Fig. 5.12).

The error signal is inputed in the feedback circuit illustrated in Fig. 5.13 that adjusts the PZT of the 844 nm diode laser to keep the laser wavelength on resonance with 422 nm atomic line. To lock the laser we reduce the scan of the 844 nm laser PZT
Figure 5.13: Electronic circuit used to lock 422 nm laser to strontium ion transition.

to zero and adjust the PZT offset (not shown) until we detected our error signal. This centers our laser close to the 422 nm ion resonance, such that it falls within the lock range of the circuit. Next, we close the servo loop by switching the servo electronic to feedback mode using the switch across the integrating stage of our network. The servo loop now locks to the 844 nm laser PZT center of the atomic transition, and the circuit gain is adjusted to get a tight lock.
Figure 5.14: System Schematic. This is an accurate layout of the 422 nm system on the optics table. Those elements labelled with PBS are polarizing beam splitters, QWP are quarter wave plates, HWP are half wave plates, and PD are photo-diodes. The AOM is an acoustic-optic modulator and the EOM is an electro-optic modulator.
Chapter 6

1092 nm Optical Repumping Laser

6.1 Repumper Laser

The intensity of the 422 nm laser is sufficient to populate the $^2D_{3/2}$ level every $\sim 200$ ns during laser cooling. This level is a metastable state with a life time of nearly 3 ms, so ions in this state will be trapped during the course of the experiment, and unable to scatter photons. The goal of rempumping is to eliminate losses to this state, $^2P_{1/2}$ to $^2D_{3/2}$ branching ratio is 1:13, by optical pumping the shelved ions back to the excited P state which spontaneously decays back to the ground state of the cooling transition.

An extended cavity diode laser was purchased to supply light at 1092 nm for rempumping. The tuning range of the laser is several Giga Hertz, and the observed drift rate is $< 300$ MHz/hr. The maximum output power of the laser is 17 mW. We use up to 15 mW of this light for optical repumping of the ions in the plasma chamber. This is a lot more power than is needed to saturate the 1092 nm transition for our experiment.

Nearly 1 mW of 1092 nm light is sent to the strontium ion discharge cell for
Figure 6.1: A plot of the 1092 nm laser absorption profile in a strontium ion discharge cell.

frequency referencing the laser. Since the $^2P_{1/2}$ to $^2D_{3/2}$ transition is not a ground state transition, we can not do saturation spectroscopy. Instead we overlap the laser with the pump beam used in the 422 nm metrology set-up (see Fig. 5.11). This causes the 1092 nm laser to interact with atoms being pumped to the D level by the 422 nm saturation spectroscopy pump beam. As a result, we obtain an absorption profile of the P-D transition.

Figure 6.1 is a plot of the 1092 nm laser absorption profile recorded on an oscilloscope. The feature near the absorption minimum is a result of the on 422 nm pump beam replacing the population in the D level that was removed with the 1092 nm laser during the scan. The width of this feature is the width of the $^2S_{1/2}$ to $^2P_{1/2}$ transition. This means that the linewidth of the 1092 nm laser will be limited to
Figure 6.2: A plot of the error signal used to lock the 1092 nm laser to the ion discharge cell.

linewidth of the 844 nm laser (8 MHz). This does not present a problem since the 1092 nm transition in the plasma cloud is power broadened by the repumper beam.

The error signal is obtained via Pound-Drever Hall technique, because side-bands are on the 422 nm pump beam. Figure 6.2 is a plot of the error signal used to lock the repumper laser PZT to the 1092 nm transition in the cell. The signal was inputed into the feedback servo shown in Fig. 6.3. The PZT to cell lock was reasonable, so we found no need for additional feedback to current modulation input. It is important to note for our set-up the 422 nm laser must be frequency stabilized in order to lock the repumper laser. This is necessary to acquire the lock point and remain in locking range of the circuit.
Figure 6.3: The feedback network to frequency stabilize 1092 nm laser.

The optical pumping experiments were done to test the 1092 nm with our ultracold neutral plasma. This was accomplished by overlapping the repumper beam with the 422 nm cooling laser in the plasma chamber. Then the fluorescence spectra was monitored for cooling beam on times of 5, 10, 15, and 20 μs after photoionization. These studies were done with and without the repumper laser to determine its effect on the $^2D_{3/2}$ population. (Figure 6.4).
Figure 6.4: Illustration of optical pumping experiments using the 1092 nm laser. The spectra is collected by turning the pump beam off for 500 ns and probing the plasma with the image beam for 1 µs.

6.2 Optical Pumping Data

Figure 6.5 is the plasma spectra with and without the repumper beam. The data with the cooling beam on and repumper on for all delay times is the same. However, the peak fluorescence decays in time for the cooling beam on and repumper off curves. This is a result of an increase D level population due to optical pumping from the cooling laser. The data from this experiment indicates that the 1092 nm laser prevents ions from being trapped in $^2D_{3/2}$ level, which is necessary for laser cooling.
Figure 6.5: This is a plot of the plasma spectra for various delay times with and without the repumper laser.
Chapter 7

Laser Cooling Studies

This chapter presents the theoretical concepts of laser cooling and describes progress made toward laser cooling an ultracold neutral plasma. The high powered laser described in chapters 4 & 5 is implemented to cool the ions. The cooling process is complicated by recombination and plasma expansion. However, this complication is removed by making the expansion slow and applying the fluorescence imaging techniques described in chapter 3. The results obtained from the preliminary cooling studies reveal that the ion velocity profile is thermalized, which is attributed to velocity changing collisions (vcc).

7.1 Laser Cooling Ion Theory in an UNP

7.2 Scattering Force

The light force on an ion/atom is due to momentum transferred when an atom absorbs a photon from a laser beam. The momentum of the ion changes by $\hbar \vec{k}$, where $\vec{k}$ is the wave vector of the incoming photon. Emission of the absorbed photon is in random directions, thus recoil momentum from emission summed over many cycles
will average to zero. As a result, the ion gains momentum in the incoming laser beam direction of travel. The resulting force is given by the following equation:

\[ F = \frac{d\vec{p}}{dt} = \hbar \vec{k} R, \quad (7.1) \]

and \( R \) is the excitation rate of the atoms. The scattering rate for an atom (two level) is given by the equation [31],

\[ R = \frac{S_0 \Gamma / 2}{1 + S_0 + [2(\Delta + \omega_D)/\Gamma]}, \quad (7.2) \]

where \( \Gamma \) is the spontaneous decay rate for the excited state. This rate depends on the laser detuning from atomic resonance \( \Delta = \omega_{\text{laser}} - \omega_{\text{atom}}, \) \( S_0 = I_{\text{laser}}/I_{\text{sat}} \) is the on-resonance saturation parameter, and the Doppler shift seen by the moving ions \( \omega_D = -\vec{k} \cdot \vec{v}. \) From Eq. 7.2 it is obvious the force is velocity dependent, and this dependence must be carefully taken into consideration to achieve optimal cooling for the ions.

Combining Eqs. 7.1 and 7.2, the expression of the scattering force is given by

\[ F_{\text{scatt}} = \hbar \vec{k} \frac{S_0 \gamma / 2}{1 + S_0 + [2(\Delta + \omega_D)/\gamma]}, \quad (7.3) \]

This is a damping force that saturates at \( \hbar \vec{k} \gamma / 2 \) and is limited by the excited state decay rate. For strontium ions this implies a maximum deceleration \( a_{\text{max}} = 7.4 \times 10^5 \), which is nearly five orders of magnitude greater than the force of gravity. This means the ion velocity can be strongly damped, and the characteristic damping time is equal
to 14.6 µs for \( |\Delta| = \gamma / 2 \). This time describes the exponential decay in energy due to the cooling force \( F = -\alpha \vec{v} \). The scattering force is significant for ions having velocities satisfying the following condition:

\[
|\Delta - \vec{k} \cdot \vec{v}| \leq \frac{\gamma}{2} \sqrt{1 + S_0}
\]  

(7.4)

The condition of Eq. 7.4 implies that atoms having a Doppler shift within one linewidth of the Lorenztian of Eq. 7.3 can be cooled. Thus, a laser beam having the proper detuning with respect to the atomic resonance frequency counter propagating with atoms having velocity \( v \) can decelerate a velocity class of atoms with a width of \( \delta v \approx \frac{\gamma}{k} \) and place them in a lower velocity group.

### 7.3 Affects of Expansion and Recombination on Ion Cooling

For successful laser cooling of the ions, plasma expansion and electron-ion recombination is crucial. After, ions equilibration they accelerate radially due to the outward pressure exerted by the electron gas. This force counteracts the deceleration of ions from photon scattering. The magnitude of this force and expansion velocity can be determined from equations given in chapter 2. For typical plasma sizes of 1 mm, the expansion force on an ion displaced \( \sigma \) from the cloud center, immersed in 10 K electrons, is equivalent to the saturating value of the scattering force. In addition, the corresponding expansion energy calculated at the characteristic damping time is 2 K.
This suggests for successful ion cooling, the electrons need to be cold in the plasma. However, cold electrons would stimulate three-body recombination (TBR) in the plasma system. Three body recombination refers to the mechanism in which an ion recombines to form a highly excited Rydberg atom, and the energy released in the process heats a second electron to conserve momentum and energy. This process was studied in detail for ultracold neutral plasmas in [32] and [33]. The rate for TBR is defined as

\[ R_{tbr} \approx 3.8 \times 10^{-21} T_e^{-9/2} n_e^2, \]  

(7.5)

and varies with electron temperature as \( T_e^{-9/2} \). Therefore, lower electron temperature yields a higher three-body recombination rate.

For 10 K electrons with a typical peak plasma density of \( 3.5 \times 10^{15} \text{ m}^{-3} \) the three-body recombination rate is \( R_{tbr} = 1.47 \times 10^6 \text{ s}^{-1} \). Therefore, on the time scale required for laser cooling, approximately 22 percent of the ions are able to recombine. This is adverse to cooling since the cooling laser would be on resonant with the ion core of the formed Rydberg atoms. As a result, transitions to the autoionizing levels would occur due to core excitation. Decays from these levels causes the formation of an ion and hot electron with energy equal to the resonance ion transition \( T_{e,hot} \), which is much higher than \( T_e \) of the thermalized electrons [34].

Unfortunately, these two factions of electrons do not thermalized due to the small electron-electron elastic cross section \( \sigma_{ee} \propto T_{e,hot}^2 \), so ions can undergo elastic collisions with these super heated electrons causing recoil-induced ion heating. The recoil en-
The ion temperature is estimated to decrease by approximately 0.5 K in 14.6 µs, and the ion heating rate due to collisions with the super hot electrons for a plasma cloud size = 1 mm and density on the order of $10^{15}$ is approximately 20 µs. Therefore, cooling the ions would be inefficient.

The discussion of the influences of expansion and TBR indicates that laser cooling is not feasible for low electron temperatures and for typical plasma sizes and densities. Initially, it appears that TBR can be eliminated by increasing the electron temperature. Unfortunately, increasing the electron temperature results in a faster expansion, and this causes the electron temperature to decrease as a result of adiabatic cooling. Fortunately, by having the ability to increase the initial size $\sigma_i$ of the plasma, TBR can be completely eliminated and the expansion can be slow.
The dependence of the electron temperature and expansion velocity on initial plasma size is illustrated in Figs. 7.1 and 7.2. In Fig. 7.1 the solid line indicates the electron temperature decreases dramatically from initial value of 48 K to 20 K in 15 µs. This places the plasma in the TBR regime. One the other hand, the dashed line, which represents $\sigma_i = 2.5$ mm, shows no significant decreases in electron temperature. Therefore, having a relatively large initial plasma size along with higher electron temperature would alleviate TBR from the system.

Figure 7.2 is a graph which depicts the effect of the plasma expansion velocity on the initial cloud size. Comparison of the solid and dash curves of the graph indicates the expansion velocity is significantly decreased for $\sigma_i = 2.5$ mm ($T_{e,initial} = 48$ K for both curves). In addition, the expansion rate, which is the inverse of characteristic plasma time $\tau_{expansion}$, is much smaller. This implies that a large plasma size could make the time scale for expansion slow enough so there is time for laser cooling.

For UNP created for laser cooling studies in this thesis, $\sigma_i \approx 2.2$ mm and $T_{e,initial} = 48$
Absorption + Spontaneous emission ≈ Net scattering force \( \vec{F} \propto -a\vec{v} \)

\[ \text{Absorption} + \text{Spontaneous emission} \approx \text{Net scattering force} \left( \vec{F} \propto -a\vec{v} \right) \]

**Figure 7.3:** An illustration of one-dimensional laser cooling for an ion with velocity \( v \).

K, to eliminate the chance for TBR to occur and slow expansion. The resulting characteristic expansion time using values is approximately 30 \( \mu s \). This time is slower but comparable to the velocity damping time of the ions. Although this implicates laser cooling is feasible, it would not be very effective going against the plasma expansion for experimental conditions in this thesis.

However, laser cooling would significantly affect ions at the cloud center where the expansion in zero. Fortunately, we can monitor these ions using the fluorescence imaging and spectroscopy techniques described in chapter 3. Using this technique, the thermal temperature associated with ions located at the center of the plasma is studied for evidence of cooling.

### 7.4 One-Dimensional Laser Cooling of a Strontium Ion

For experiments presented in this thesis, one-dimensional laser cooling is implemented, by illuminating the plasma cloud with two linearly polarized laser beams.
Figure 7.4: Net scattering force as a function of velocity for various detunings. The force is normalized with $\hbar \vec{k} \gamma / 2$. The velocity is expressed in units of most probable velocity $V_{mp}$ for a 1.2 K ion temperature.

propagating in the opposite directions as illustrated in Fig. 7.3. This specific configuration of beams is necessary to interact with ions traveling in both directions. The two beams act independently, and the net scattering force on an ion moving with velocity $v$ in the cloud is given by the following expression:

$$F_{net} = \hbar \vec{k} S_0 \left( \frac{\gamma / 2}{1 + S_0 + [2(\Delta - \omega_D)/\gamma]} + \frac{\gamma / 2}{1 + S_0 + [2(\Delta + \omega_D)/\gamma]} \right). \quad (7.6)$$

Figure 7.4 is a plot of equation 7.5, the net scattering force, as a function of velocity for detunings $\Delta$ equal to $-3\gamma / 2, \gamma, -\gamma / 2$. The velocity is expressed in units of most probable velocity $V_{mp}$, and $S_0 = .90$, which is congruent to the value used in the cooling experiments. The linear portion of the curves illustrate that the net force is approximately linear for small velocities, which can be expressed in the following equation

$$F_{net} \approx \alpha \vec{v} \approx \frac{8\hbar k^2 S_0 \Delta}{\gamma(1 + S_0 + (2\Delta/\gamma)^2)^2}, \quad (7.7)$$
where $\alpha$ represents the damping coefficient, which is equivalent to the slope of the curves. The three curves suggest that the best detuning for optimal damping of the ion velocity is $\Delta = -\gamma/2$.

The maximum capture velocity for the net scattering force is determined by equating the Doppler shift equal to the laser detuning from the transition: $v_{\text{capture}} \approx \frac{\Delta}{k}$. For strontium ions and the optimal detuning mentioned above, $\frac{\Delta}{k} \sim 4.3$ m/s, which is one-third of $V_{mp}$. The fraction of ions in one direction of the 1.2 K ion thermal distribution inside the velocity range $-\frac{\Delta}{k}$ to $\frac{\Delta}{k}$ is approximately 34 percent. Thus, enough ions can be cooled to cause changes in the thermal temperature at the cloud center.

### 7.5 Spectroscopy Studies of a Laser Cooled Plasma

The laser cooling studies are performed by illuminating the plasma cloud with approximately 40 mW of collimated 422 nm laser light that is red-detuned $-\gamma/2$ from resonance, immediately after photoionization. The laser detuning is achieved by passing the beam through a 200 MHz center frequency AOM, since the 422 nm was locked 250 MHz below resonance. The counter-propagating beam is generated by retro-reflection of the incoming beam using a mirror. The size of the cooling beam has $1/e^2$ radius equal to 5.25 mm, which corresponds to $S_0 = .90$. The duration of the cooling beam is controlled by shuttering the RF power to the AOM.

The repumper beam is co-propagating with the laser cooling beam, and is kept continuously on throughout the experiment to prevent population of the $^2D_{3/2}$ state.
Figure 7.5: The Fluorescence spectra of laser cooled Sr ions for 5 µs after photoionization. The spectra is analyzed in the cloud center and over entire cloud. The data indicates we are able to obtain the ion thermal temperature.

The intensity of the repumper beam is 67 mW/cm² (1/e² radius is 3.5 mm), which is six times the effective saturation intensity. The repumper laser is tuned on resonance with the 1092 nm transition.

The laser cooled ion spectra is collected by turning the cooling beam off for 500 ns and probing the plasma with the image beam for 1 µs. The image beam is detuned using a 200 MHz center frequency AOM, and it can be detuned ± 60 MHz from the strontium ion transition. The image beam power is 10 mW with 1/e² radius of approximately 0.6 cm. This intensity is much less than $I_{sat}$ for the $^2S_{1/2} - ^2P_{1/2}$ transition. This is necessary to prevent optical pumping to the D state while taking spectra.

Figure 7.5 is the fluorescence spectra for the laser cooled ions for 5 µs after photoionization with the repumper on. Figures 7.5(a) and 7.5(b) are the spectra analyzed for the center region, having x-y region of interest as .05σ(t)$\approx$4.82×10⁴ and entire plasma cloud $\approx$1.45×10⁷. From the data the effective temperature
Figure 7.6: The plasma spectra in the cloud center for 5 and 20 $\mu$s after photoionization, for the cases of no cooling and cooling.

$T_{i,\text{eff}}$, is smaller in the center region. In this region the expansion velocity is zero thus the effective ion temperature is equal to the thermal temperature, $T_{i,\text{therm}}$, of the ions defined in Eq. 2.1. This illustrates our ability to remove the effects of expansion from our studies and monitor the thermal temperature.

The chart 7.6 illustrates the center cloud spectra for 5 and 20 $\mu$s after photoionization, for the cases of no cooling and cooling. These data sets show no variation in the thermal temperature or narrowing of the doppler width, for the cooling laser up to 20 $\mu$s. Obviously, no cooling is taking place inside the center of the plasma cloud. This spectra suggests that the ions are not scattering many photons, which is counter to optical pumping experiments in chapter 6 and cooling calculations done in a previous section.

To further investigate the implications of the data sets analyzed in Fig. 7.6 the
Figure 7.7: Laser cooling spectra in the cloud center with and without repumper for 5 \(\mu\)s, after photoionization. The results are interesting because there are no lamb dips in the spectra.

Figure 7.8: This is an representation of the effect of vccs on optical pumping of the ions in the plasma. The figure to the left illustrates atoms removed from a ground state velocity bin by the cooling laser. The picture to the right illustrates vccs replenishing the resonant velocity bin with atoms not yet pumped from other velocity bins. This would occur if the vcc rate is greater than or equal to optical pumping rate.
spectra of the laser cooled ions were studied with and without the repumper. Figure 7.7 is the ion laser cooled spectra with out the repumper 20 µs after photoionization and \( \delta = -\gamma / 2 \). These results are interesting since the cooling laser should cause dips in the spectra corresponding to population of the \( ^2D_{3/2} \) level. However, this would occur if the ions were experiencing velocity changing collisions (vcc), which would replenish the resonant velocity bin with ions from another group not yet pumped.

VCCs are completely elastic collisions between ions of different velocity groups. These collisions become important for cooling studies, when the collision rate is comparable to the optical pumping rate. If the collision rate was much smaller than the optical pumping rate, the ions would scatter many photons and cool before a collision would occur. When the two rates are similar, exchanges between the velocity bins effectively prevent cooling from occurring on the time scale of the experiments in this thesis. Since, these velocity exchanges cause the plasma velocity profile to thermalize.
Chapter 8

Collision Rates

For two particles colliding in a plasma the collision cross section is given by

$$\sigma = \int (1 - \cos \theta) d\sigma. \quad (8.1)$$

The quantity $\theta$ is the scattering angle, $d\sigma = 2\pi \rho d\rho$ is the differential scattering cross section, and $\rho$ is the impact parameter. If the interaction potential and kinetic energy are comparable (close encounters), the principal contribution to Eq. 8.1 is from large scattering angles. The collision cross section in this situation is given by the following expression [35]

$$\sigma = \pi R_c^2, \quad (8.2)$$

where, $R_c = \frac{e^2}{2\pi \epsilon_0 \mu u_{thermal}}$ is the distance for which the Coulomb interaction is equal to the kinetic energy. The quantities $\mu$ and $u_{thermal}$ are the reduced mass and relative velocity respectively. The collision rate $\nu_c$ is obtained by substituting Eq. 8.2 into
Figure 8.1: Scattering in a plasma. $R_c$ is the distance of closest approach, $\theta$ is the scattering angle, and $\rho$ is the impact parameter.

The expression $\nu_c = \sigma u_{\text{thermal}} n_i$ to get:

$$\nu_c = \pi \left( \frac{e^2}{2\pi \epsilon_0 \mu u_{\text{thermal}}} \right)^2 n_i u_{\text{thermal}}. \quad (8.3)$$

The collision rate in Eq. 8.3 is a poor approximation. The reason being, this equation assumes no long range coulomb interaction. In fact, it turns out that the number of collisions due to long range Coulomb interactions (distant encounters) is much greater than number of close collisions, and the cumulative effect of these collisions outweighs that of collision resulting in large scattering angles [36]. Therefore, a valid approximation for the scattering cross section is given by [37],

$$\sigma = \int (1 - \cos \theta) 2\pi \rho d\rho \approx \int \theta^2 \pi d\rho = \frac{e^2}{4\pi \epsilon_0 \mu u_{\text{thermal}}} \int \frac{d\rho}{\rho}. \quad (8.4)$$

The integral diverges in the limit of small and large impact parameters. The divergence at small impact parameter violates the assumption that the principal con-


Figure 8.2: Plot of the spitzer collision rate and ion plasma frequency $\omega_{pi}/2\pi$ versus plasma density. The graph indicates the spitzer collision rate formula becomes invalid when the density is high and temperature low. As the density increase, the spitzer rate approached $\omega_{pi}/2\pi$ before the $\ln \Lambda$ diverges.

Divergence is from small scattering angles. The divergence at large impact parameter is due to the long range of the coulomb potential. These divergences are removed by introducing $\rho_{\min} = R_c$ and $\rho_{\max} = \lambda_D$ (Debye screening length) as lower and upper limit cut-offs. As a result, the scattering cross section can be written as:

$$\sigma = \pi \left(\frac{e^2}{2\pi \epsilon_0 \mu_{\text{thermal}}}\right)^2 \ln \Lambda \quad (8.5)$$

Historically, the removal of these divergences was done by Spitzer and a more thorough explanation can be found in [38].
The term $\ln \Lambda$ is the famous Coulomb logarithm, and $\Lambda$ may be written as the ratio between the Debye length and Coulomb length $\Lambda = \frac{\lambda_D}{\lambda_c}$. In terms of the coupling parameter $\Gamma$, the logarithm may be written as $\ln \Lambda \approx \ln \left( \sqrt{\frac{3}{\Gamma}} \right)^{\frac{3}{2}}$. Figure 8.2 is a log-log plot of the Spitzer ion-ion collision rate versus plasma density for various ion temperatures. This expression diverges when the temperature and density correspond to a strongly coupled plasma, $\Gamma > 1$. As a result, this graph indicates that calculating the collision rate using the Spitzer formula is not valid for our plasma, which have densities between $10^{14} \text{ - } 10^{15} \text{ m}^{-3}$ and ion temperature approximately 1 K.

The break down in the Spitzer formula occurs because the Debye screening length becomes similar to an interparticle spacing. As a result, the ion interaction is limited to their closest neighbor, which is counter to the assumption of Spitzer’s equation that distance encounters are the primary contribution to the collision cross section. The close encounter collision rate given in Eq. 8.3 is also invalid because it was derived assuming only two-body encounters. This may not apply for strongly coupled plasmas, which do not behave as a gas. On the other hand, Fig. 8.2 illustrates that the Spitzer rates become similar to the ion plasma frequency as conditions become favorable for a strongly coupled system. In addition, the behavior of $\omega_{pi}^{-1}$ is unperturbed for $\Gamma > 1$. Is the ion plasma frequency a more suitable rate for collisions in a strongly coupled plasma?

The results from the above section suggests that tradition model for ion-ion collisions in plasma systems are invalid for ultracold neutral plasmas. This make sense,
since in strongly coupled plasmas, the Coulomb interaction dominates, and the ions tend to behave more like a fluid. Their positions are correlated and the ions tend to move only an interparticle spacing. The interactions in this regime can be visualized as ions colliding with their nearest neighbors. We know that these type of collisions cause the ions kinetic energy to oscillate [6]. As the ions oscillate in their local potential wells, energy and momentum is transferred between interacting neighbors. Since energy and momentum is conserved in the ion subsystem, this is equivalent to velocities exchanges between them.

The maximum change in the ion velocity, as a result of this type of collision, is close to the ion thermal velocity. This could explain the thermalization of the velocity profile observed in the optical pumping and cooling experiments. We will show that the time scale for these collisions/oscillations is the inverse plasma ion frequency $\omega_{pi}^{-1}$. This further supports the conclusions of Fig. 8.2, which suggests that the plasma ion frequency $\omega_{pi}/2\pi$ is a possible collision rate.

8.1 Collisions

The purpose of this section is to present the theory that can be used to simulate the cooling spectra for very short times after photoionization without the 1092 nm repumper laser and incorporating velocity changing collisions. It is beyond the scope of this thesis to provide the tools for an in-depth collision study. However, it does provide enough theory to model optical pumping spectra of strontium ions undergoing
velocity changing collisions to identify the thermalization time in a strongly coupled plasma.

The basic approximations that will be used in considering vcc collisions are: 1) The vccs do not cause excitation or de-excitation of the ions in the plasma. This is valid for relatively low plasma temperatures used in our laser cooling studies. 2) The collisions occur instantaneously, with respect to decay and optical pumping rates. This rules out atoms decaying when a collision occurs, which supports the first approximation. 3) The vcc collisions fall in the regime of the strong collision approximation, which implies that the probability that an ion having a specific velocity after a collision is unrelated to its velocity before the collision. Thus, ions experiencing vcc are assumed to have a thermal distribution [39].

8.2 Rate Equations

Using the above approximations, the rate equations for the i-th level incorporating the effect of velocity changing collisions can be written as,

$$\frac{dN_i(v_k, t)}{dt} = \frac{dN_{i,\text{optical}}(v_k, t)}{dt} + \frac{dN_{i,\text{collisions}}(v_k, t)}{dt},$$  \hspace{1cm} (8.6)

where $N_{i,\text{optical}}(v_k, t)$ is the i-th level population from the k-th velocity bin, and it contains the usual optical pumping, stimulated emission, and decay rate terms.
\(N_{i,\text{collisions}}(v_k, t)\) is the collision term and is given by,

\[
\frac{dN_{i,\text{collisions}}(v_k, t)}{dt} = -\omega_{\text{collision}}N_i(v_k, t) + \omega_{\text{collision}}f(v_k)\eta_i(t),
\tag{8.7}
\]

where \(f(v)\) is the normalized one-dimensional Maxwell-Boltzman distribution function, and \(\eta_i = \int N_i(v) dv\) is the total population in the i-th level. The first collision term, \(-\omega_{\text{collision}}N_i(v_k, t)\), describes the loss from k-th group population in the i-th level. The second collision term, \(\omega_{\text{collision}}f(v_k)\eta_i(t)\), describes the contribution to k-th group population in i-th level from all other velocity groups. Equation 8.7 is a basic collision model that functions to preserve total particle number and Maxwell Boltzman velocity distribution of the ions.

The fluorescence spectra can be modeled by solving the above rate equations for strontium ions. It should be noted that this model does not include the effects of coherences, two-photon or Raman type processes. However, for time scales of interest, these effects can be neglected. In addition, these equations do not incorporate the effect of laser cooling. Therefore they are invalid for long time scales.

### 8.3 Three Level Strontium Ion Rate Equations

The coupled rate equations for the three level ions system are:

\[
\begin{align*}
\dot{N}_1 &= -R_{12}N_1 + (W_{21} + \Gamma_{21})N_2 + \Gamma_{31}N_3 - \omega_{\text{coll.}}N_1 + \omega_{\text{coll.}}f(v_k)\eta_1(t), \\
\dot{N}_2 &= R_{12}N_1 - (W_{21} + \Gamma_{21} + \Gamma_{23} + W_{23})N_2 + R_{23}N_3 - \omega_{\text{coll.}}N_2 + \omega_{\text{coll.}}f(v_k)\eta_2(t),
\end{align*}
\]
\[ \dot{N}_3 = (\Gamma_{23} + W_{23+})N_2 - (R_{23} + \Gamma_{31})N_3 - \omega_{\text{coll.}}N_3 + \omega_{\text{coll.}}f(v^k)\eta_3(t), \] (8.8)

where the subscripts 1, 2, and 3 denote the S, P, and D levels. As discussed in the previous section these equations can be interpreted in terms of their no collisions and collision contributions. For example, the equation for \( N_1 \) may be interpreted as follows: The first term describes loss due to optical pumping, the second term is a contribution due to spontaneous and stimulated emission from level 2, the third term includes gain from the spontaneous decay of level 3, and finally the fourth and fifth terms represent the effects of collisions.

The laser optical pumping rate, \( R_{ij} \), and stimulated emission rate, \( W_{ij} \), are equivalent to "Fermi’s Golden Rule", and are defined in Eq. 7.2. The terms \( \Gamma_{21}, \Gamma_{23}, \) and \( \Gamma_{31} \) are the transition rates from the levels. We also impose that S-S, P-P, S-P, and P-D collision rates are the same, since the collisions are mediated by the long range Coulomb interaction.

**Figure 8.3:** This figure is a plot of the numerical solution to the coupled rate equations in Eq. 8.8 for the cases of no collision (left) and velocity changing collisions (right). The cooling laser was on for 1 \( \mu \)s, repumper is off, and the laser power is 30 mW. The laser detuning was \( \Delta = -\gamma/2 \). Note the two clear Lamb dips in the \( N1 \) plot on the left.
The coupled rate equations in Eq. 8.8 are solved using numerical methods for differential equations [40]. For each time step the population $N_1$, $N_2$, and $N_3$ are obtained for each velocity bin. Figure 8.3 is a plot of the numerical solution to the coupled rate equations for the cases of no collision (left) and velocity changing collisions (right). The cooling laser was on for 1 µs, repumper was off, and the laser power and detuning were 30 mW and $\Delta=-\gamma/2$, respectively. On the right, we set $\omega_{coll.}=\omega_{pi,max}$, where $\omega_{pi,max}$ is the plasma ion frequency corresponding to the peak density ($n_o=2 \times 10^{14} \text{ m}^{-3}$). These results of the simulation indicate that collisions occurring at a rate of $\omega_{pi,max}$ are capable of thermalizing the plasma cloud.

8.4 Determination of Thermalization Rate

The fluorescence signal of our experiment is proportional to the populations ($N_1+N_2$) of the simulation, because optical pumping spectra is taken after the pump laser is off for 500 ns. As a result, the population in the excited N2 level decays to N1. We compare the simulations to experimental data to check the model and extract upper and lower bounds on the thermalization rate. Figure 8.4 is a plot of the fluorescence spectra taken for laser cooling beam on for 0 µs, 0.5 µs, and 0.9 µs with zero detuning and repumper laser off for an ultracold neutral plasma. The peak plasma density is $2 \times 10^{14} \text{ m}^{-3}$, which corresponds to $\omega_{pi,max} = 2.54 \times 10^5 \text{ s}^{-1}$. The scale factor to compare the simulated points to the data is obtained by the dividing the peak of the data with the peak of the simulation at 0 µs cooling time. We
Figure 8.4: Fluorescence spectra taken for laser cooling beam on for 0 µs, .5 µs, and .9 µs. The laser is on resonance with the principle Sr ion transition. The repumper laser is off for each time. The scale factor to compare the simulated points to the data is obtained by the dividing the peak of the data with the peak of the simulation at 0 µs cooling time. The laser detuning is zero and the laser power was 20 mW. We set $\omega_{\text{coll}} = 4\omega_{\text{pi,max}}$. The temperature used in the simulation is 1.6 K.

show this figure primarily to indicate that our optical pumping parameters are correct.

The laser power and center frequency were not fit parameters. They were measured experimentally.

By comparing the simulation points with the data for various collision rates we can show that $4\omega_{\text{pi,max}}$ is the collision rate that best models the optical pumping data.

Fig. 8.5 is a comparison of the 0.5 µs data with the simulation for collision frequencies $4\omega_{\text{pi,max}}$, $1\omega_{\text{pi,max}}$, $\frac{1}{10}\omega_{\text{pi,max}}$, and $\frac{1}{100}\omega_{\text{pi,max}}$. As the collision frequency decreases the simulation does not fit the data well. From this comparison, we set a lower limit on
Figure 8.5: This is a plot of the of simulation points for various collision frequencies compared to spectra taken at .5 µs. The circles represent the data and solid lines are the simulation points taken at $4\omega_{\text{pi, max}}$, $\omega_{\text{pi, max}}$, $\frac{1}{10}\omega_{\text{pi, max}}$, and $\frac{1}{100}\omega_{\text{pi, max}}$.

the collision frequency to be $4\omega_{\text{pi, max}}$.

Similarly, by comparing the experimental data using values larger than $4\omega_{\text{pi, max}}$, we can obtain an upper limit on the thermalization rate. In Fig. 8.6 we compared the data with simulation collision frequencies $3\omega_{\text{pi, max}}$, $5\omega_{\text{pi, max}}$, $7\omega_{\text{pi, max}}$, and $10\omega_{\text{pi, max}}$. As the collision frequency increases the peak of the simulation data becomes larger than the peak of my fluorescence data. Since, we know $4\omega_{\text{pi, max}}$ is the best value, a conservative upper limit on the collision frequency could be $5\omega_{\text{pi, max}}$. We did the same for the 0.9 µs data as illustrated in Fig. 8.7 and obtained the same results.
Figure 8.6: A plot of the simulation points for various collision frequencies compared to spectra taken at .5 µs. The circles represent the data and solid lines are the simulation points taken at $3\omega_{pi,max}$, $5\omega_{pi,max}$, $7\omega_{pi,max}$, and $10\omega_{pi,max}$. From this comparison we obtained an upper limit on the thermalization rate.

Figure 8.7: This is a plot of the simulation points for various collision compared to spectra taken with at .9 µs.
Chapter 9

Conclusion

In conclusion I would like to say that we have identified that velocity changing collisions are occurring in our plasma. We have learned from fluorescence spectroscopy that the collisions thermalize the plasma cloud. This prevented us from being able to laser cool the ions in our plasma. To investigate these collisions we did optical pumping studies in the plasma cloud, and developed a collisional model to simulate the data. The collision model incorporated velocity bins population changes into the rate equations. This model was based on two criterion: conservation of particle number and preservation of the Maxwell Boltzman distribution of the particles velocities at all times. The rate equations including additional terms from our collision model successfully reproduced the data from optical pumping experiments. From a fit of the simulation to the data, we determined the collision rate for strongly coupled ions in a neutral plasma to be on the order of the plasma ion frequency.
Appendix A

422 nm Ring Cavity Autocad Drawings

This section contains the autocad drawings of the 844 nm ring-cavity that was constructed in the machine shop at Rice. The figures below are the crystal housing and mounting attachments, the optical bread board base, and the lid to cover the ring cavity respectively.
Figure A.1: This is an autocad drawing of the ppKTP crystal housing and mounting attachments.
Figure A.2: This is an autocad drawing for the 844 nm ring cavity optical bread board.
Figure A.3: This is an autocad drawing for the 844 nm ring cavity optical bread board lid.
Appendix B

Code to Simulate Spectra

%code numerically solves the rate equations for a three level
%It takes into account collisions.

clear all;
close all;
timepoints=[1,1251,2251];
wpifactor=[4,1,1/10,1/100];
for j=1:3 %this loop runs for different optical pumping times
    for q=1:4 %this loop runs for various ion plasma frequency
        htimestep=4*10^(-10);
        numberofpointstime=timepoints(j);%point are 12501(.5us),22501(.9us),37501(1.5us)
        Ibluesat=(114*10^(-3))*(100/1)^(2);
        Pbluelaser=20*10^(-3);
        bluebeamsize=.5*10^(-2);
        Ibluelaser=(2*Pbluelaser)/(pi*(bluebeamsize)^(2));
\[ I_{irsat} = (1 \times 10^{-3}) \times (100/1)^2; \]

\[ P_{laser} = 20 \times 10^{-3}; \]

\[ \text{irbeamsize} = 0.2 \times 10^{-2}; \]

\[ \text{repumperstatus} = 0; \]

\[ I_{laser} = \text{repumperstatus} \times \left( \frac{(2 \times P_{laser})}{\pi \times (\text{irbeamsize})^2} \right); \]

\[ \text{stimulatedemissionstatus} = 1; \]

\[ s_{12} = \frac{I_{bluelaser}}{I_{bluesat}}; \quad \% \text{saturation parameter of S-P level} \]

\[ s_{23} = \frac{I_{laser}}{I_{irsat}}; \quad \% \text{saturation parameter for P-D level} \]

\[ \gamma_{21} = 2 \times \pi \times 20.0 \times 10^6 \times (\sqrt{1 + s_{12}}); \]

\[ \gamma_{23} = 2 \times \pi \times 1.6 \times 10^6 \times (\sqrt{1 + s_{23}}); \]

\[ \gamma_{31} = 2 \times \pi \times 0.33 \times 10^3; \]

\[ \text{detuningpurple} = 0; \]

\[ \text{detuningir} = 0; \]

\[ \lambda_{purple} = 421.7 \times 10^{-9}; \]

\[ k_{purple} = \frac{2 \times \pi}{\lambda_{purple}}; \]

\[ \lambda_{air} = 1092 \times 10^{-9}; \]

\[ k_{ir} = \frac{2 \times \pi}{\lambda_{air}}; \]

\[ T_{thermal} = 1.6; \]
m = 87.905 \times (1.66054 \times 10^{-27})

kb = 1.381 \times 10^{-23}

vtherm = \sqrt{\frac{kb \times T_{thermal}}{m}}

Te = 48

delaytime = 15 \times 10^{-6}

noi = (0.5) \times 4.00 \times 10^{14}

\sigma_i = 2.31 \times 10^{-3}

charexpansiontime = \sqrt{m \times (\sigma_i)^2 / (kb \times (Te + T_{thermal}))}

\sigma_t = \sqrt{\sigma_i^2 \times (1 + (delaytime)^2 / (charexpansiontime^2))}

\sigma_{tx} = \sqrt{\sigma_i^2 \times (1 + (delaytime)^2 / (charexpansiontime^2))}

\sigma_{ty} = \sqrt{\sigma_i^2 \times (1 + (delaytime)^2 / (charexpansiontime^2))}

\sigma_{tz} = \sqrt{\sigma_i^2 \times (1 + (delaytime)^2 / (charexpansiontime^2))}

totalnumberofions = 3.92 \times 10^{3}

e = 1.6022 \times 10^{-19}

\epsilon = 8.8542 \times 10^{-12}

\nu = wpifactor(q) \times \sqrt{noi \times e^2 / (m \times \epsilon)}; \text{collision frequency}

\text{upperlimitv} = 5.0
vthermlimit=upperlimitv*vtherm;

numberofvelocitysubintervals=100;%number of subintervals

%size of velocity bin
velocitybin=(vthermlimit-(-vthermlimit))/numberofvelocitysubintervals;

for l=1:(numberofvelocitysubintervals)
    velocityleft(l)=-vthermlimit+((l-1)*velocitybin);%(left-hand endpoints)
    velocityright(l)=-vthermlimit+((l)*velocitybin);%(right-hand endpoints)
end

for d=1:(numberofvelocitysubintervals+1)%end points in each velocity bin
    velocityendpoint(d)=-vthermlimit+((d-1)*velocitybin);
end

for c=1:numberofvelocitysubintervals
    %midpoint in each velocity bin
    velocity(c)=(velocityendpoint(c)+velocityendpoint(c+1))/2;
end

velocitydims=size(velocity);

numberofpointsvel=velocitydims(1,2);%I reassign the number of velocity points
%numerical integrating M-B distribution number in bin

tol=1*10^(-6);%numerical integration tolerance

for p=1:numberofpointsvel
    F=@(v)totalnumberofions*sqrt(m/(2*pi*kb*Tthermal))*exp(-m*(v).^(2)/(2*kb*Tthermal));
    Q(p)=quad(F,velocityleft(p),velocityright(p),tol);
end

numberofionsvelbin=Q(:);

for k=1:numberofpointsvel%loop solves rate equation in each velocity bin

%pumping and stimulated emission rate for S-P transition

R12(k,1)=(s12*(gamma21)/2)*
( ((1+s12+(2*(detuningpurple-(kpurple*velocity(k)))/gamma21).^2).^(-1)
  +(1+s12+(2*(detuningpurple+(kpurple*velocity(k)))/gamma21).^2).^(-1));

W12(k,1)=(s12*(gamma21)/2)*
( ((1+s12+(2*(detuningpurple-(kpurple*velocity(k)))/gamma21).^2).^(-1)
  +(1+s12+(2*(detuningpurple+(kpurple*velocity(k)))/gamma21).^2).^(-1));

%pumping and stimulated emission rate for P-D transition

R23(k,1)=(s23*(gamma23)/2)*
\[
\frac{1}{1 + s_{23} + \left(\frac{2(\text{detuning}_{IR} - \text{kir} \cdot \text{velocity}(k))}{\gamma_{23}}\right)^2} + \frac{1}{1 + s_{23} + \left(\frac{2(\text{detuning}_{IR} + \text{kir} \cdot \text{velocity}(k))}{\gamma_{23}}\right)^2}.
\]

\[W_{23}(k,1) = \left(\frac{s_{23} \cdot \gamma_{23}}{2}\right) \left(\frac{1}{1 + s_{23} + \left(\frac{2(\text{detuning}_{IR} - \text{kir} \cdot \text{velocity}(k))}{\gamma_{23}}\right)^2} + \frac{1}{1 + s_{23} + \left(\frac{2(\text{detuning}_{IR} + \text{kir} \cdot \text{velocity}(k))}{\gamma_{23}}\right)^2}\right)\]

\[N_1 = \text{numberofions}_{velbin}(k); \%	ext{each bin}\]

\[N_2 = 0;\]

\[N_3 = 0;\]

\[t_0 = 0;\]

\[
\text{for } i = 1: \text{numberofpointstime} \%	ext{loop for each time step}\]

\[
\text{if } i == 1, \text{factor} = 0;\]

\[
\text{else}\]

\[
\text{factor} = 1;\]

\[
\text{end}\]

\%	ext{rate equations no collisions}\]

\[N_1 = N_1 + \text{factor} \cdot h_{\text{time step}} \cdot (-R_{12}(k) \cdot N_1 + W_{12}(k) \cdot N_2 + \gamma_{21} \cdot N_2 + \gamma_{31} \cdot N_3);\]

\[f = N_1 - \text{factor} \cdot (h_{\text{time step}} \cdot (-R_{12}(k) \cdot N_1 + W_{12}(k) \cdot N_2 + \gamma_{21} \cdot N_2 + \gamma_{31} \cdot N_3));\]
N2 = N2 + factor * h timestep * (R12(k) * f - W12(k) * N2 - gamma21 * N2 - gamma23 * N2 - W23(k) * N2 + R23(k) * N3);

g = N2 - factor * h timestep * (R12(k) * f - W12(k) * N2 - gamma21 * N2 - gamma23 * N2 - W23(k) * N2 + R23(k) * N3);

N3 = N3 + factor * h timestep * (gamma23 * g + W23(k) * g - R23(k) * N3 - gamma31 * N3);

time = (t0 + (i - 1) * h timestep);

P1(i, k) = N1;
P2(i, k) = N2;
P3(i, k) = N3;
T(i, 1) = time;

end
end

matrix = size(P1(:, :)); \% dims of population matrix:

\% rows of P1 matrix imply time and column imply velocity.

\% Thus P1(i, k) means the N1 population for ith time and kth velocity.

dimsrow = matrix(1, 1); \% the number of rows in P1
dimscol = matrix(1, 2); \% the number columns in P1

numpointspectra = dimsrow; \% number of time steps
spectra(:,q,j)=(P1(dimsrow,:)+P2(dimsrow,:)); % total spectra vector

if max(P1(dimsrow,:)) > max(P3(dimsrow,:)) % determine text location for figure
  ygreat=max(P1(dimsrow,:));
else
  ygreat=max(P3(dimsrow,:));
end

xleast=min(velocity(:));

% figures below are population, no collisions

figure(1)
plot(T(:,P1(:,dimscol),'r-');
hold on
plot(T(:,P2(:,dimscol),'g.-');
hold on
plot(T(:,P3(:,dimscol),'b-');
hold off
figure(2)

title('Level Population vs. Velocity (no collisions)')

plot(velocity(:),P1(dimsrow,:),’r.’);
hold on
plot(velocity(:),P2(dimsrow,:),’go’);
hold on
plot(velocity(:),P3(dimsrow,:),’bd’);
hold off

xlabel('velocity [m/s]');
ylabel('Population [Ions]');

title('Level Population vs. Velocity (no collisions)');

text(xleast,ygreat,['time is =',num2str(max(T(:)))]);

legend('N1','N2','N3','NorthEastOutside');


% solve rate equation with collisions

% M-B distribution for T_{thermal}=1.6 K at each velocity midpoint

MBfuntion=sqrt(m/(2*pi*kb*T_{thermal}))*exp(-m*(velocity).^(2)/(2*kb*T_{thermal}));

Q1=transpose(P1);
Q2=transpose(P2);
Q3=transpose(P3);

%this is a sum over all velocity bin for i-th level for each time step
collisionterm2Slevel=transpose(sum(velocitybin*Q1));
collisionterm2Plevel=transpose(sum(velocitybin*Q2));
collisionterm2Dlevel=transpose(sum(velocitybin*Q3));

for m=1:numberofpointsvel
%loop solves rate equation in each velocity bin

%pumping and stimulated emission rate for S-P transition
R12(m,1)=(s12*(gamma21)/2)*
((1+s12+(2*(detuningpurple-(kpurple*velocity(m)))/gamma21)^2)^(-1)+
(1+s12+(2*(detuningpurple+(kpurple*velocity(m)))/gamma21)^2)^(-1));
W12(m,1)=(s12*(gamma21)/2)*
((1+s12+(2*(detuningpurple-(kpurple*velocity(m)))/gamma21)^2)^(-1)+
(1+s12+(2*(detuningpurple+(kpurple*velocity(m)))/gamma21)^2)^(-1));

%pumping and stimulated emission rate for P-D transition
R23(m,1)=(s23*(gamma23)/2)*
((1+s23+(2*(detuningir-(kir*velocity(m)))/gamma23)^2)^(-1)+
(1+s23+(2*(detuningir+(kir*velocity(m)))/gamma23)^2)^(-1));
\begin{align*}
W_{23}(m,1) &= \frac{s_{23}(\gamma_{23})}{2} \left( (1 + s_{23} + (2(\text{detuning}_\text{ir} - (k\text{ir}_\text{r} \times \text{velocity}(m))/\gamma_{23})^2)^{-1} +
\right.
onumber \\
&\quad \left. (1 + s_{23} + (2(\text{detuning}_\text{ir} + (k\text{ir}_\text{r} \times \text{velocity}(m))/\gamma_{23})^2)^{-1} \right) 
\end{align*}

\begin{align*}
N_1 &= \text{numberofions}\_\text{velbin}(m) \\
N_2 &= 0 \\
N_3 &= 0 \\
t_0 &= 0 \\
\end{align*}

\begin{verbatim}
for n=1:numberofpointstime
    if n==1,
        factorc=0;
    else
        factorc=1;
    end

    % rate equations with collisions
    N1=N1+factorc*htimestep*(-R12(m)*N1+W12(m)*N2+gamma21*N2+gamma31*N3+
        (-wpi*N1+wpi*MBfuntion(m)*collisionterm2Slevel(n)));
\end{verbatim}
\begin{verbatim}
f=N1-factorc*(htimestep*(-R12(m)*N1+W12(m)*N2+\gamma_{21}N2+\gamma_{31}N3+
(-w_{pi}N1+w_{pi}MBfuntion(m)*collisionterm2Slevel(n))));
N2=N2+ factorc*htimestep*(R12(m)*f-W12(m)*N2-\gamma_{21}N2-\gamma_{23}N2-
W23(m)*N2+R23(m)*N3+(-\gamma_{23}+ w_{pi}MBfuntion(m)*collisionterm2Plevel(n)));
g=N2-factorc*htimestep*(R12(m)*f-W12(m)*N2-\gamma_{21}N2-\gamma_{23}N2-
W23(m)*N2+R23(m)*N3+(-\gamma_{23}+ w_{pi}MBfuntion(m)*collisionterm2Plevel(n)));
N3=N3+factorc*htimestep*(\gamma_{23}g+W23(m)*g-R23(m)*N3-\gamma_{31}N3+
(-\gamma_{31}+w_{pi}MBfuntion(m)*collisionterm2Dlevel(n)));

Ctime=(t0+(n-1)*htimestep);
CP1(n,m)=N1;
CP2(n,m)=N2;
CP3(n,m)=N3;
CT(n,1)=time;

end
end

matrix=size(CP1(:,:));
dimsrow=matrix(1,1);
dimscol=matrix(1,2);
\end{verbatim}
numpointspectra=dimsrow;

cspectra(:,q,j)=CP1(dimsrow,:)+CP2(dimsrow,:);

if max(CP1(dimsrow,:)) > max(CP3(dimsrow,:))
ygreat=max(CP1(dimsrow,:));
else
  ygreat=max(CP3(dimsrow,:));
end
xleast=min(velocity(:));

figure(3)
plot(velocity(:),CP1(dimsrow,:),'r.');
hold on
plot(velocity(:),CP2(dimsrow,:),'go');
hold on
plot(velocity(:),CP3(dimsrow,:),'bd');
hold off
xlabel('velocity [m/s]');
ylabel('Population [Ions]');
title('Level Population vs. Velocity (collisions)');
text(xleast,ygreat,['time is =',num2str(max(CT(:)))]);
legend('N1','N2','N3','NorthEastOutside');
set(v,'Interpreter','none');

figure(4)
plot(velocity(:),spectra(:),'go');
xlabel('velocity [m/s]');
ylabel('Population [Ions]');
title('Spectra vs. Velocity (no collisions)');

text(xleast,ygreat,['time is =',num2str(max(CT(:)))]);
legend('N1(no collisions)+N2(no collisions)','NorthEastOutside');
set(v,'Interpreter','none');

figure(5)
plot(velocity(:),cspectra(:),'go');
xlabel('velocity [m/s]');
ylabel('Population [Ions]');
title('Spectra vs. Velocity (collisions)');

text(xleast,ygreat,['time is =',num2str(max(CT(:)))]);
legend('N1(collisions)+N2(collisions)','NorthEastOutside');
set(v,'Interpreter','none');
% reads data files and compares to collision simulation

[filenames] = textread('experimentdatabatchfile.txt', '%q', 'commentstyle', 'matlab');
dimensionsfiles = size(filenames);
numberoffiles = dimensionsfiles(1,1);
for u = 1:numberoffiles, % loops through all data files
data = dlmread(char(filenames(u)),'\t'); % read data
detuning = data(:,1);
dflourescencedata = data(:,2);
dimensionsofdata = size(data(:,1));
numberofdatapoints = dimensionsofdata(1,1);
span = 1; % this smooths the data
window = ones(span,1)/span;
smoothedflourdata = convn(florescencedata(:,1), window, 'same');
flourescencesignal(:,u) = smoothedflourdata(:,1);
laserdetuningarray(:,u) = detuning(:,1);
end
end
scalefactor = max(flourescencesignal(:,1))/max(cspectra(:,1,1));
% scale factor
scaledcspectra = scalefactor*cspectra(:,:,1);

cutstart = 20;
cutend = 80;
cutvelocity = velocity(cutstart:cutend);
cutscaledcspectra = scaledcspectra(cutstart:cutend,:,:);

% laser frequency to velocity conversion

laservelocity = (2*pi*laserdetuningarray(:,:,1)*10^-6)/kpurple;
Bibliography


